AD-A071 649 DAYTON UNIV OHIO F/G 20/11

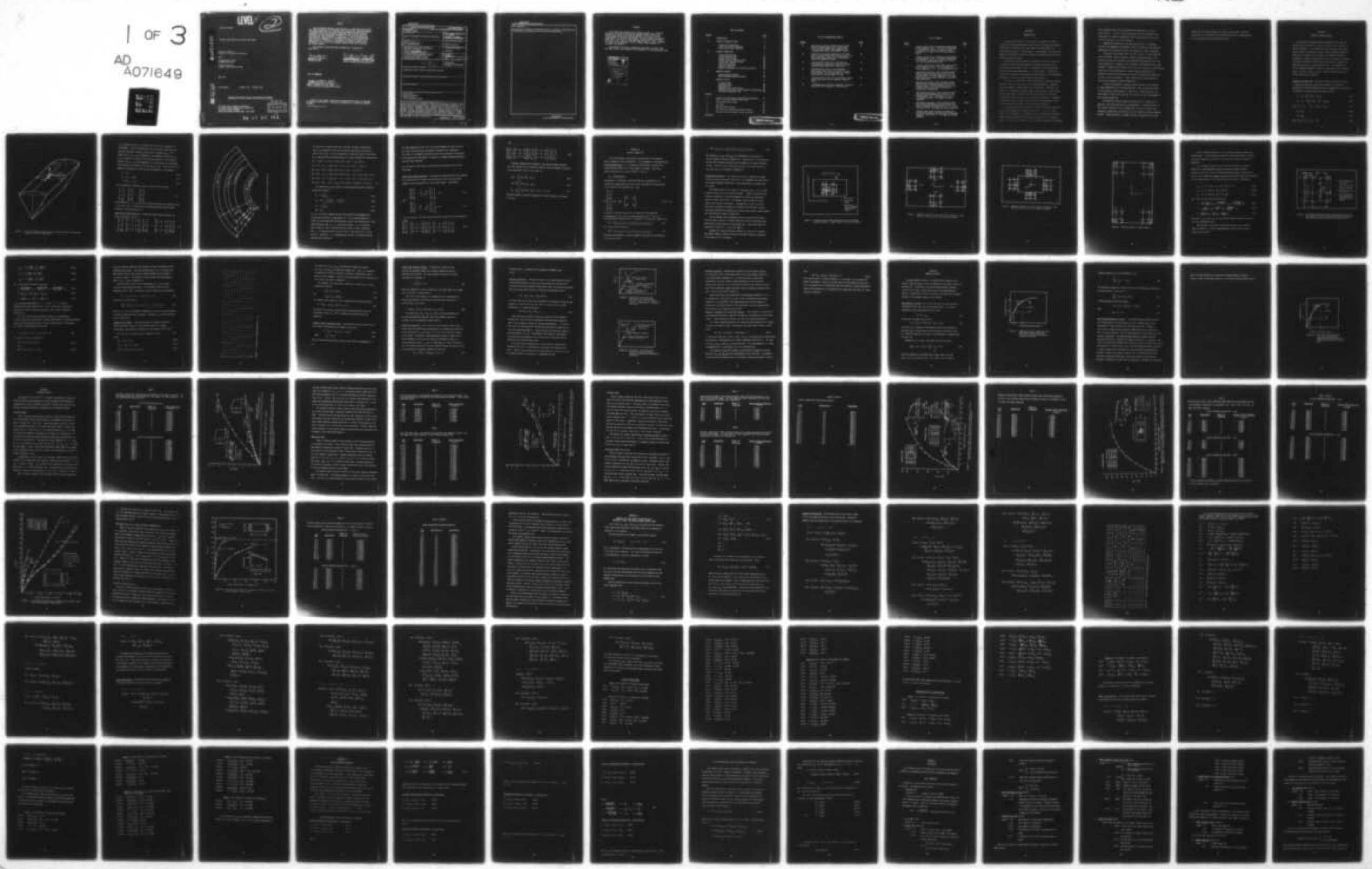
COLLAPSE LOAD ANALYSIS FOR PLATES AND SHELLS.(U)

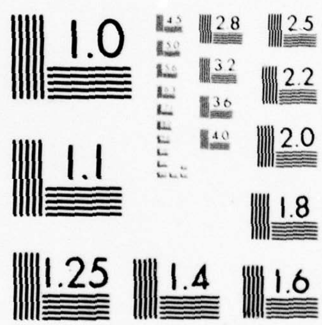
MAY 79 N R BAULD, K SATYAMURTHY F33615-76-C-3145

UNCLASSIFIED AFFDL-TR-79-3038 NL

1 OF 3

AD A071649





MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS 1963-A

LEVEL II

2

B.S.

AFFDL-TR-79-3038

AD A071649

COLLAPSE LOAD ANALYSIS FOR PLATES AND PANELS

Nelson R. Bauld, Jr.
Professor of Engineering Mechanics

and

Kailasam Satyamurthy
Graduate Assistant

Clemson University
Clemson, South Carolina 29631

MAY 1979

DDC FILE COPY

Final Report October 1977 - October 1978

Approved for public release; distribution unlimited.

DDC
RECEIVED
JUL 24 1979
D

AIR FORCE FLIGHT DYNAMICS LABORATORY
AIR FORCE WRIGHT AERONAUTICAL LABORATORIES
AIR FORCE SYSTEMS COMMAND
WRIGHT-PATTERSON AIR FORCE BASE, OHIO 45433

79 07 23 101

NOTICE

When Government drawings, specifications, or other data are used for any purpose other than in connection with a definitely related Government procurement operation, the United States Government thereby incurs no responsibility nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data, is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use, or sell any patented invention that may in any way be related thereto.

This technical report has been reviewed and is approved for publication.

Narendra Khot

NARENDRA S. KHOT
Project Engineer

Hartley M. Caldwell

HARTLEY M. CALDWELL, III, Capt, USAF
Chief, Analysis and Optimization Br.

FOR THE COMMANDER

Ralph L. Kuster Jr.

RALPH L. KUSTER, JR., Col, USAF
Chief, Structures and Dynamics Division

Copies of this report should not be returned unless return is required by security considerations, contractual obligations, or notice on a specific document.

AIR FORCE/56780/5 July 1979 — 210

19

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER 18 AFFDL-TR-79-3038	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER 9
4. TITLE (and Subtitle) 6 COLLAPSE LOAD ANALYSIS FOR PLATES AND SHELLS		5. TYPE OF REPORT & PERIOD COVERED Final Report Oct 1977 - Oct [redacted]78
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) 10 Nelson R. Bauld, Jr. Kailasam Satyamurthy		8. CONTRACT OR GRANT NUMBER(s) 15 F33615-76-C-3145
9. PERFORMING ORGANIZATION NAME AND ADDRESS Department of Mechanical Engineering University of Clemson Clemson, ~~North Carolina~~ 29631 S.C		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 2307-N5-01
11. CONTROLLING OFFICE NAME AND ADDRESS AF Flight Dynamics Laboratory (FBRA) Structures and Dynamics Division Wright-Patterson AFB, OH 45433		12. REPORT DATE 11 May [redacted]79
		13. NUMBER OF PAGES 209 12 217p.
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) 16 2307 17 05		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Buckling Load
Composite Structures
Plates and Shells
Finite Difference Methods

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report presents a theoretical foundation, computational procedures, and a computer program for the analyses of bifurcation or collapse of circular, cylindrical panels and rectangular plates. Discretization is accomplished via a variable-spacing, orthogonal, finite-difference grid using the principle of total potential energy. The modified Newton-Raphson iterative procedure is used to generate nonlinear load-deflection curves and a separate branch of the program provides a bifurcation analysis based on linear, prebuckling behavior. The program is capable of handling rectangular cut-out and initial geometric imperfections; a variety of zero force and zero displacement boundary conditions; and

79 07 23 101

20. Continued

point loads, line loads, and distributed loads in transverse, longitudinal and circumferential directions, as well as axial end-compression.

FOREWORD

This report was prepared by Dr. Nelson R. Bauld, Jr., Visiting Scientist in the Design and Analysis Methods Group, Analysis and Optimization Branch, Structures and Dynamics Division, Air Force Flight Dynamics Laboratory (AFFDL/FBR), Air Force Wright Aeronautical Laboratories, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio. This work was performed under Project Nr. 2307, "Research in Flight Vehicle Dynamics," Task Nr. 230705, "Basic Research in Structures and Dynamics." 61102F

The technical report was released by the Author in October 1978. The report covers work conducted from September 1977 through October 1978.

Accession For	
NTIS GRA&I	☒
DDC TAB	☐
Unannounced	☐
Justification	
By	
Distribution/	
Availability Codes	
Dist	Avail and/or special
A	

TABLE OF CONTENTS

PRECEDING PAGE BLANK

LIST OF ILLUSTRATIONS (CONT'D)

PRECEDING PAGE BLANK

LIST OF TABLES

SECTION I

INTRODUCTION

The buckling behavior of circular, cylindrical, fiber-reinforced, laminated panels under general loading and boundary conditions is characterized by either the appearance of a bifurcation point or a limit point. Accordingly, an effective numerical procedure should be capable of detecting bifurcation points and limit points.

Bifurcation points can be preceded by either a linear or a nonlinear relationship between the applied load and the resulting displacements, while a nonlinear relationship must always precede the appearance of a limit point. Therefore, an effective numerical procedure should allow for the nonlinear behavior of the load-displacement relationship.

Circular, cylindrical, fiber-reinforced, laminated panels exhibit either of these types of buckling behavior depending upon the manner in which they are constructed. Material properties of individual layers, fiber orientations, layer thicknesses, and layer locations within the stack, all interact to determine the character of the buckling behavior. Boundary conditions, type of load, and location of concentrated loads also influence the nature of the buckling behavior.

A material characterization of fiber-reinforced laminates has been presented by several writers [1, 2, 3, 4]. Each layer of the laminate is assumed to be an orthotropic material whose material properties are known relative to its natural axes. Strains are assumed to vary linearly through the laminate

in accordance with the Kirchhoff-Love hypothesis of thin shell theory. Stress-strain relations for the laminate are developed via these assumptions and the definitions for the stress-resultants for the laminate.

Unbalanced laminates are characterized by a coupling between the membrane and moment stress-resultants and balanced laminates are characterized by the absence of this coupling.

Gallagher and Padlog [5], Vos [6], Lien [7], and Lang [8] have presented finite-element, energy-based procedures for characterizing the buckling and postbuckling behavior of shell type structures. Bushnell [4], Bushnell and Almroth [10], and Bushnell, Almroth and Brogan [11] have led the way in developing effective finite-difference, energy-based procedures for characterizing the buckling and initial postbuckling behaviors of shell type structures. Their efforts have culminated in the general purpose program STAGS [12].

A finite-difference, energy-based procedure was adopted as the foundation for the generation of the system equations in the present investigation. Details of the procedure are explained as they arise during the course of the development of the system equations.

The Modified Newton-Raphson method [13] is employed to generate the load-deflection relationship. A modification of the linear equation solver SESØL [14] is used to solve the algebraic equations associated with the Modified Newton-Raphson method. Modifications of SESØL include incorporation of the

capability of calculating the system determinant, and the capability of factoring the coefficient matrix independently of load-reduction and backsubstitution.

SECTION II

THEORY OF COMPOSITE PANELS

Fiber-reinforced composite panels considered in this study are assumed to be constructed by stacking individual layers of fibers on top of one another. It is assumed that each individual layer can be characterized as an orthotropic material whose orthotropic properties relative to its material axes are known. An isotropic material is considered as a special case where each layer of the stack is isotropic. The angular orientation of the fibers of each layer is assumed to be known with respect to a common reference axis. The reference axis for the present development is taken as a generator of the cylindrical panel. The angle θ shown in Figure 1 locates the fiber direction of a layer relative to a generator.

Kinematic Considerations. The composite panel consisting of N layers is required to obey the Kirchhoff-Love hypothesis. Accordingly, appropriate strain-displacement relations for moderately large rotations of tangents to the panel reference surface are [15]:

$$e_x = U_{,x} + \frac{1}{2}\phi_x^2 + \frac{1}{2}\phi^2 - W_{o,x}\phi_x \tag{1-a}$$

$$e_y = V_{,y} + W/R + \frac{1}{2}\phi_y^2 - \frac{1}{2}\phi^2 - W_{o,y}\phi_y \tag{1-b}$$

$$2e_{xy} = V_{,x} + U_{,y} + \phi_x\phi_y - W_{o,y}\phi_y - W_{o,y}\phi_x \tag{1-c}$$

$$K_x = \phi_{x,x} \tag{1-d}$$

$$K_y = \phi_{y,y} \tag{1-e}$$

$$2K_{xy} = 2K_{yx} = \phi_{y,x} + \phi_{x,y} + \phi/R \tag{1-f}$$

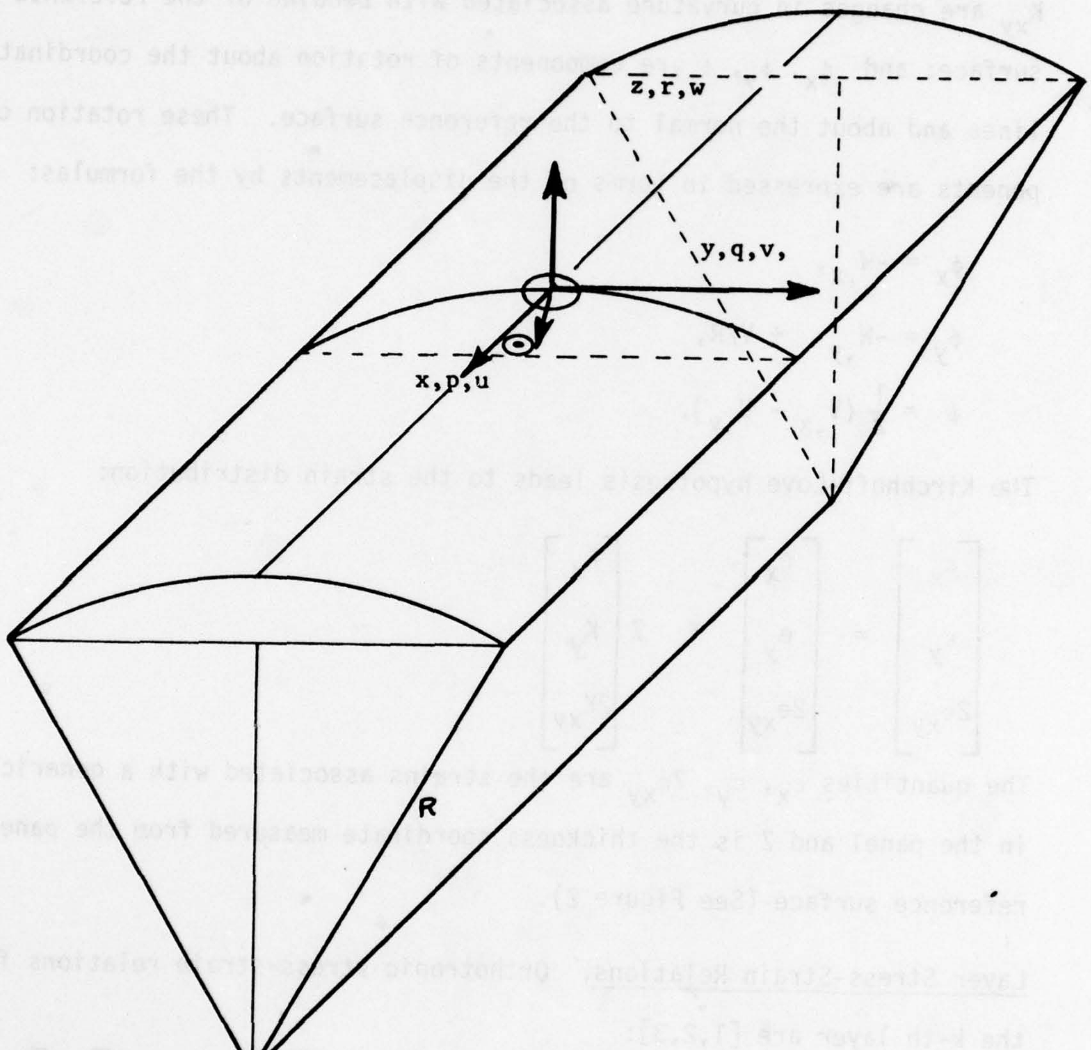


Figure 1. Circular cylindrical panel depicting coordinate system, displacement components, and load components.

U, V, W denote the axial, circumferential, and normal components of displacement. Initial imperfections are restricted to those that are normal to the panel reference surface and are denoted by W_o. The quantities e_x, e_y, e_{xy} are components of the membrane strains; K_x, K_y, K_{xy} are changes in curvature associated with bending of the reference surface; and ϕ_x, ϕ_y, ϕ are components of rotation about the coordinate lines and about the normal to the reference surface. These rotation components are expressed in terms of the displacements by the formulas:

$$\phi_x = -W_{,x}, \tag{2-a}$$

$$\phi_y = -W_{,y} + V/R, \tag{2-b}$$

$$\phi = \frac{1}{2}(V_{,x} - U_{,y}). \tag{2-c}$$

The Kirchhoff-Love hypothesis leads to the strain distribution:

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ 2\varepsilon_{xy} \end{bmatrix} = \begin{bmatrix} e_x \\ e_y \\ 2e_{xy} \end{bmatrix} + Z \begin{bmatrix} K_x \\ K_y \\ 2K_{xy} \end{bmatrix} \tag{3}$$

The quantities ε_x, ε_y, $2\varepsilon_{xy}$ are the strains associated with a generic point in the panel and Z is the thickness coordinate measured from the panel reference surface (See Figure 2).

Layer Stress-Strain Relations. Orthotropic stress-strain relations for the k-th layer are [1,2,3]:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}_k = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{21} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{61} & \bar{Q}_{62} & \bar{Q}_{66} \end{bmatrix}_k \begin{bmatrix} e_x \\ e_y \\ 2e_{xy} \end{bmatrix} + z \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{21} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{61} & \bar{Q}_{62} & \bar{Q}_{66} \end{bmatrix}_k \begin{bmatrix} K_x \\ K_y \\ 2K_{xy} \end{bmatrix} \tag{4}$$

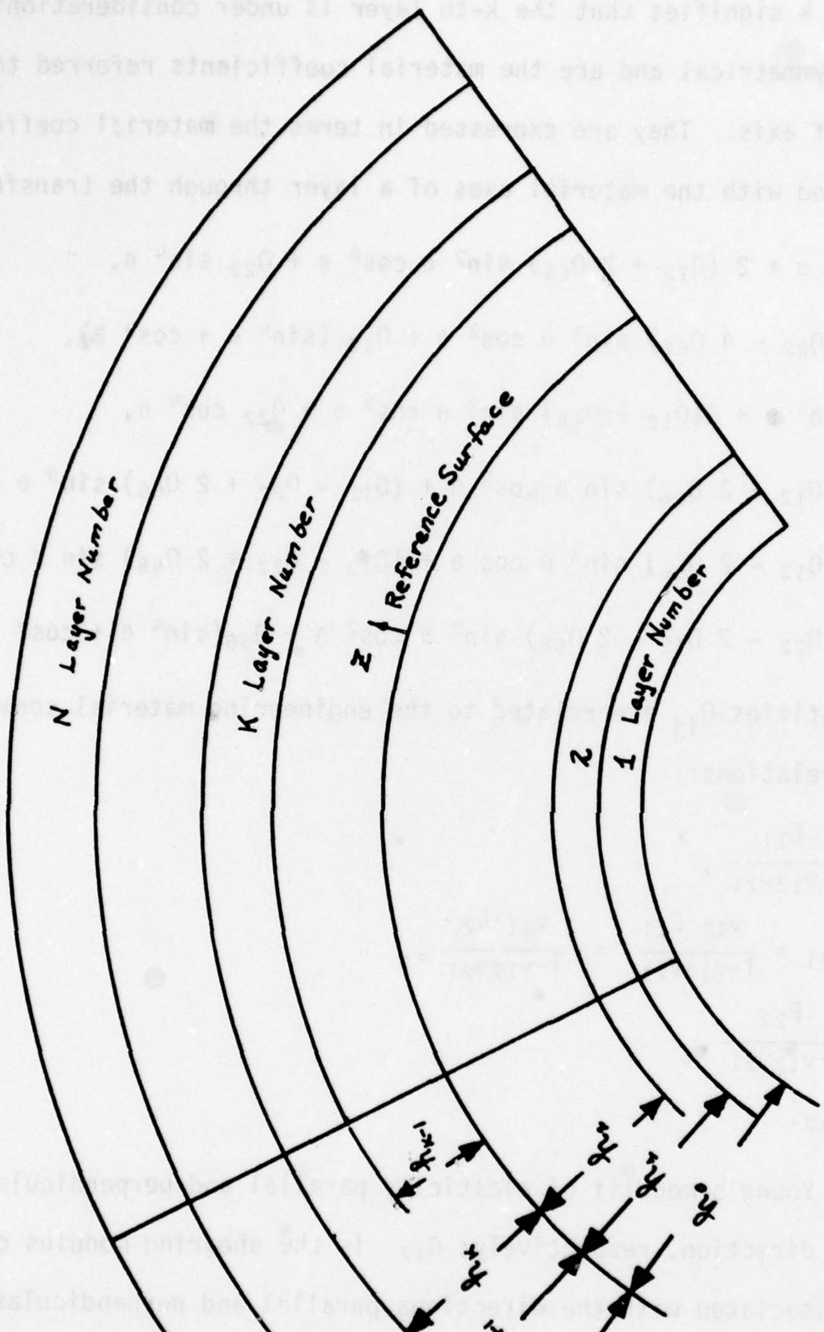


Figure 2. Cross section of composite panel.

The subscript k signifies that the k-th layer is under consideration. The $\bar{Q}_{ij}$ are symmetrical and are the material coefficients referred to a generic set of axis. They are expressed in terms the material coeffcients, Q_{ij}, associated with the material axes of a layer through the transformation:

$$\bar{Q}_{11} = Q_{11}\cos^4 \theta + 2 (Q_{12} + 2 Q_{66}) \sin^2 \theta \cos^2 \theta + Q_{22} \sin^4 \theta,$$

$$\bar{Q}_{12} = (Q_{11} + Q_{22} - 4 Q_{66}) \sin^2 \theta \cos^2 \theta + Q_{12} (\sin^4 \theta + \cos^4 \theta),$$

$$\bar{Q}_{22} = Q_{11} \sin^4 \theta + 2(Q_{12} + 2Q_{66}) \sin^2 \theta \cos^2 \theta + Q_{22} \cos^4 \theta,$$

$$\bar{Q}_{16} = (Q_{11} - Q_{12} - 2 Q_{66}) \sin \theta \cos^3 \theta + (Q_{12} - Q_{22} + 2 Q_{66}) \sin^3 \theta \cos \theta,$$

$$\bar{Q}_{26} = (Q_{11} - Q_{12} - 2 Q_{66}) \sin^3 \theta \cos \theta + (Q_{12} - Q_{22} + 2 Q_{66}) \sin \theta \cos^3,$$

$$\bar{Q}_{66} = (Q_{11} + Q_{22} - 2 Q_{12} - 2 Q_{66}) \sin^2 \theta \cos^2 \theta + Q_{66}(\sin^4 \theta + \cos^4 \theta). \quad (5)$$

The quantities Q_{ij} are related to the engineering material constants through the relations:

$$Q_{11} = \frac{E_{11}}{1-\nu_{12}\nu_{21}}, \quad (6\text{-a})$$

$$Q_{12} = Q_{21} = \frac{\nu_{12} E_{11}}{1-\nu_{12}\nu_{21}} = \frac{\nu_{21} E_{22}}{1-\nu_{12}\nu_{21}}, \quad (6\text{-b})$$

$$Q_{22} = \frac{E_{22}}{1-\nu_{12}\nu_{21}}, \quad (6\text{-c})$$

$$Q_{66} = G_{12}. \quad (6\text{-d})$$

E_{11}, E_{22} are Young's modulii of elasticity parallel and perpendicular to the fiber direction, respectively; G_{12} is the shearing modulus of elasticity associated with the directions parallel and perpendicular to the fiber direction; and ν_{12}, ν_{21} are Poisson ratios. ν_{12} is associated with a strain in the 1- direction due to a stress in the 2- direction and ν_{21} is associated with a strain in the 2- direction due to a stress in the 1- direction. It is convenient to let the 1- direction coincide with the fiber direction.

The angle appearing in Eqs. (5) is the angle between the fiber direction of a layer and the surface coordinate line parallel to a generator. This angle is considered to be positive when the coordinate line parallel to the generator of the panel is located in a counter-clockwise direction from the fiber direction.

Q_{ij} are material coefficients associated with the material axes of the k-th layer.

Panel Stress-Strain Relations. The normal and shearing stress distributions on a cross section of a panel are replaced with statically equivalent membrane forces and couples as in thin shell theory. Accordingly,

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \sum_{k=1}^{N} \int_{h_{k-1}}^{h_k} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}_k dz \,, \tag{7-a}$$

and

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \sum_{k=1}^{N} \int_{h_{k-1}}^{h_k} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}_k z \; dz. \tag{7-b}$$

Eqs. (4) and Eqs. (7) lead to the constitutive relations for the composite panel:

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} e_x \\ e_y \\ 2e_{xy} \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} K_x \\ K_y \\ 2K_{xy} \end{bmatrix} \tag{8-a}$$

and

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} e_x \\ e_y \\ 2\,e_{xy} \end{bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} K_x \\ K_y \\ 2\,K_{xy} \end{bmatrix} \tag{8-b}$$

Formulas connecting the elements of the panel stiffness matrices [A], [B], and [D] with the material properties and the geometric locations of the individual layers of the stack are:

$$A_{ij} = \sum_{k=1}^{N} \overline{Q}_{ij}{}^{k} \left(h_k - h_{k-1}\right), \tag{9-a}$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^{N} \overline{Q}_{ij}{}^{k} \left(h_k^2 - h_{k-1}^2\right), \tag{9-b}$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^{N} \overline{Q}_{ij}{}^{k} \left(h_k^3 - h_{k-1}^3\right), \; (i,j = 1,\, 2,\, 6). \tag{9-c}$$

Pertinent geometric quantities appearing in these relations are defined in Figure 2.

SECTION III
DISCRETE FORMULATION

A finite-difference, energy-based discretization of the composite panel is adopted for the investigation. Its development is considered below.

Energy Considerations. It is assumed that the reference surface of the panel has been divided into a finite number of elements. The strain energy associated with a typical element is taken as

$$2U = [\tilde{\varepsilon}]^T[\tilde{N}][\tilde{\varepsilon}]*\text{Area}. \tag{10}$$

The notation $(\tilde{\ })$ signifies a centroidal quantity, consequently, $[\tilde{\varepsilon}]$ and $[\tilde{N}]$ are, respectively, the strain vector and material stiffness matrix at the centroid of the element area. Thus,

$$[\tilde{\varepsilon}] \equiv \begin{bmatrix} e_x \\ e_y \\ 2e_{xy} \\ k_x \\ k_y \\ 2k_{xy} \end{bmatrix} \quad \text{and} \quad [\tilde{N}] \equiv \begin{bmatrix} [A] & [B] \\ [B] & [D] \end{bmatrix} \tag{11-a, 11-b}$$

In general, the strain vector $[\tilde{\varepsilon}]$ is a function of the centroidal displacements $\tilde{U}$, $\tilde{V}$, $\tilde{W}$, the first order derivatives of U, V, W, and the second order derivatives of W. Accordingly, one may write, symbolically

$$[\tilde{\varepsilon}] = f(U\ U_{,x},\ U_{,y},\ V\ V_{,x},\ V_{,y},\ W\ W_{,x},\ W_{,y},\ W_{,xx},\ W_{,xy},\ W_{,yy}). \tag{12}$$

Let a vector [d] be defined as:

$$[\tilde{d}]^T = [\tilde{U}\ \tilde{U}_{,x},\ \tilde{U}_{,y},\ \tilde{V}\ \tilde{V}_{,x},\ \tilde{V}_{,y},\ \tilde{W}\ \tilde{W}_{,x},\ \tilde{W}_{,y},\ \tilde{W}_{,xx},\ \tilde{W}_{,xy},\ \tilde{W}_{,yy}], \tag{13}$$

then the strain energy of a typical element of the panel can be expressed in the indicial form

$$2U = a_{ij}\,\tilde{d}_i\,\tilde{d}_j + a_{ijk}\,\tilde{d}_i\,\tilde{d}_j\,\tilde{d}_k + a_{ijk\ell}\,\tilde{d}_i\,\tilde{d}_j\,\tilde{d}_k\,\tilde{d}_\ell . \qquad (14)$$

The tensors a_{ij}, a_{ijk}, and $a_{ijk\ell}$ are independent of the vector $\tilde{d}_i$ and are assumed to have been symmetrized. Equation (14) is an expression for the strain energy associated with an element of the panel reference surface. Details of the calculations leading to the coefficients a_{ij}, a_{ijk}, and $a_{ijk\ell}$ are presented in Appendix A.

Finite-Difference Grid. The reference surface is assumed to be mapped onto the xy-plane as shown in Figure (3). An orthogonal grid that allows for variable spacing in both the x- and y-directions is inscribed upon this plane.

The element whose strain energy is represented by Eq. (14) is shown shaded in Figure (3) for an interior element. Elements contiguous with external boundaries or internal boundaries at cutouts are modified as shown in Figures (4) and (5). The elements cannot account for small areas at the corners of the panel. Consequently, in order to avoid a weak coupling between the degrees of freedom associated with a corner grid point and those of neighboring grid points four special corner elements are incorporated as shown in Figure (6).

For any element, nine surrounding points in the finite-difference grid are used to express a function value and its first and second order derivatives at the centroid of the element area. These grid points are depicted in Figures (3) - (5) by the symbol x.

Backward and forward difference formulas are used for all boundary and corner elements so that fictitious grid points outside the boundary of the panel are not introduced.

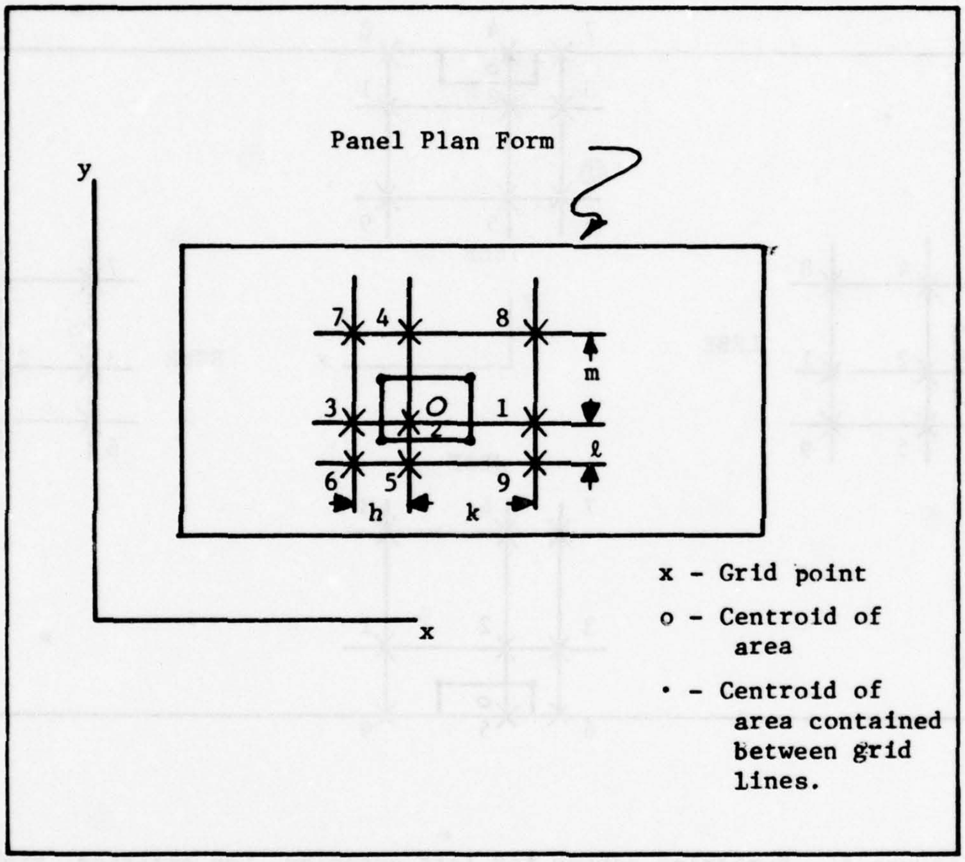


Figure 3. Variable spacing finite-difference grid for the panel reference surface. Number scheme for interior elements.

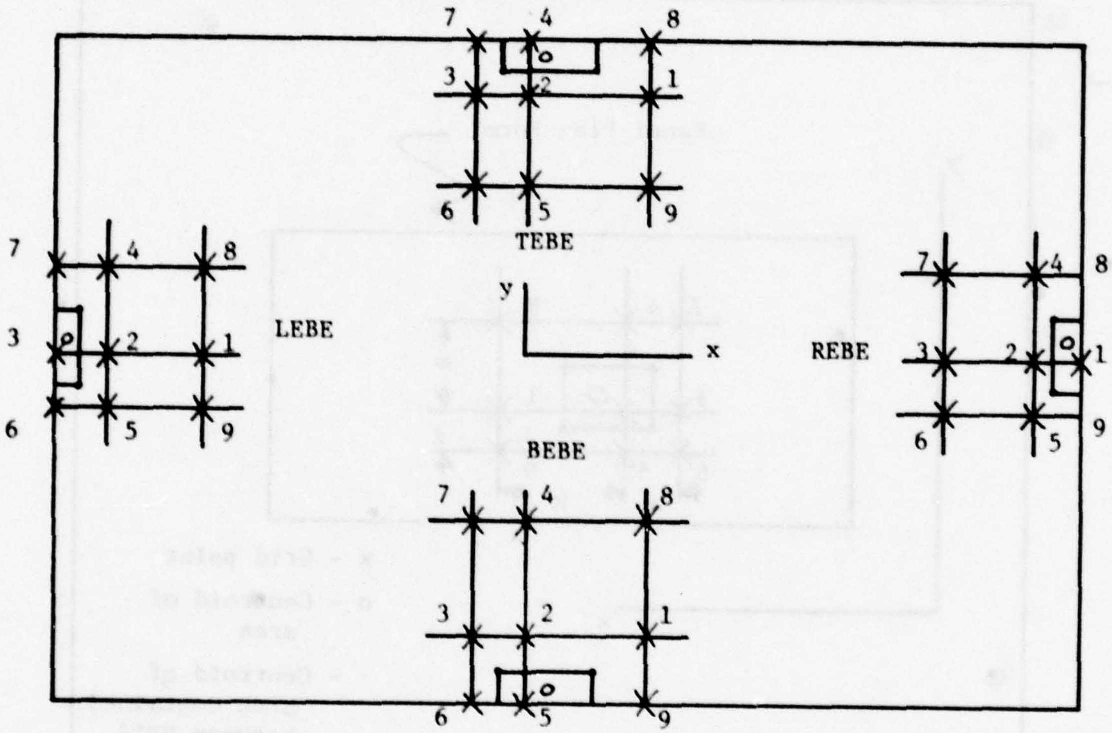


Figure 4. Numbering scheme for left exterior, top exterior, right exterior, and bottom exterior boundary elements.

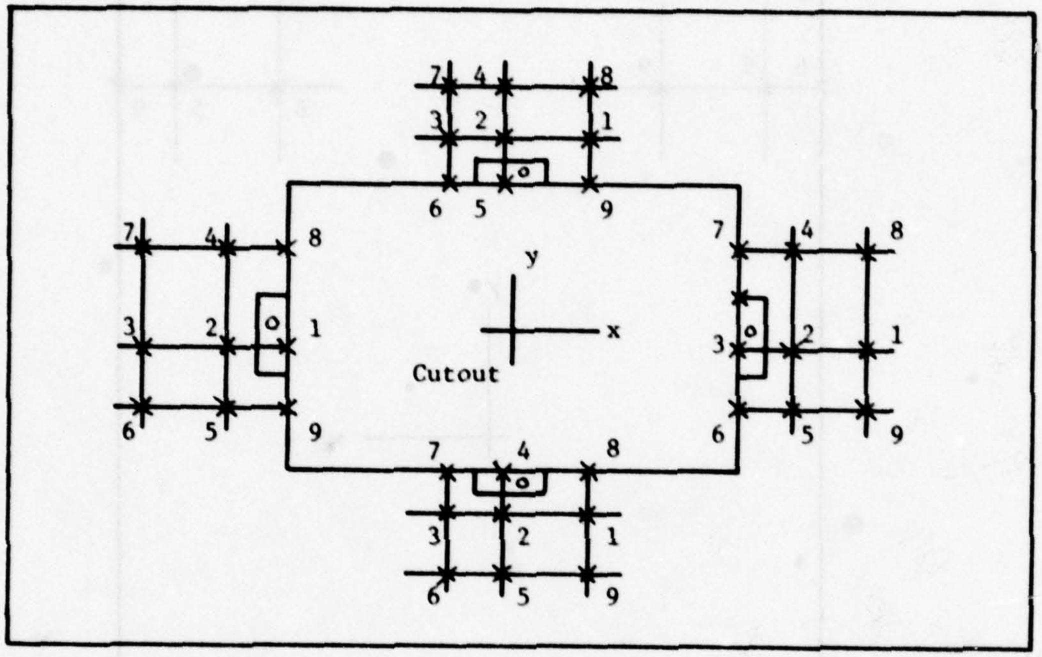


Figure 5. Numbering scheme for left interior, top interior, right interior, and bottom interior boundary elements.

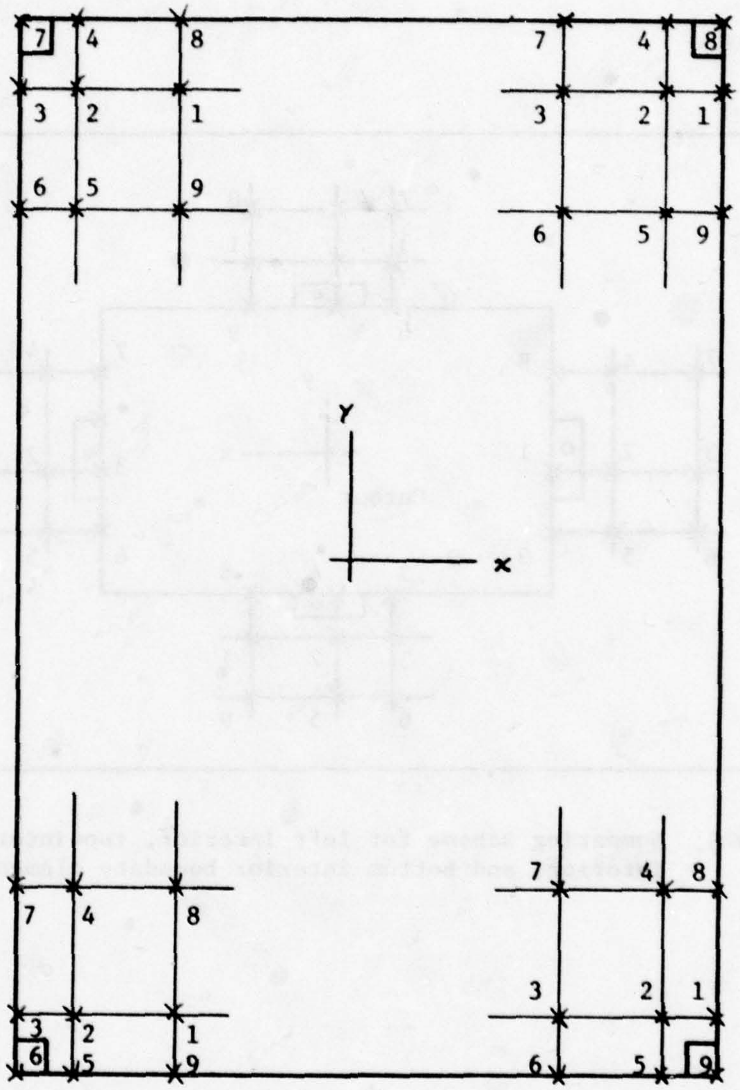


Figure 6. Numering scheme for corner elements.

Finite-difference formulas for a left external boundary element are derived below. Finite-difference formulas for the other external, internal, and corner elements are obtained in a similar manner and are recorded in Appendix B.

For an element located at the left exterior boundary as shown in Figure (7), a one-dimensional Taylor series may be used to relate a function value and its first and second derivatives along a line parallel to the x-axis through the element centroid with discrete values of the function at the intersections of this line with the y-grid lines. Accordingly,

$$f_{i+2} = \tilde{f} + \tilde{f}' \, (3/4\, h + k) + \frac{1}{2} \tilde{f}'' \, (3/4\, h + k)^2, \qquad (16\text{-a})$$

$$f_{i+1} = \tilde{f} + \tilde{f}' \, (3/4\, h) + \frac{1}{2} \tilde{f}'' \, (3/4\, h)^2, \qquad (16\text{-b})$$

$$f_{i} = \tilde{f} - \tilde{f}' \, (1/4\, h) + \frac{1}{2} \tilde{f}'' \, (1/4\, h)^2. \qquad (16\text{-c})$$

Eqs. (16) yield the centroidal quantities:

$$\tilde{f} = -\frac{3}{16} \frac{h^2}{k(h+k)} f_{i+2} + \frac{1}{4} \frac{(k+3/4h)}{k} f_{i+1} + \frac{3}{4} \frac{(k+3/4h)}{(h+k)} f_i, \qquad (17\text{-a})$$

$$\tilde{f}' = -\frac{h}{2k(h+k)} f_{i+2} + \frac{(k+\frac{1}{2} h)}{hk} f_{i+1} - \frac{(k+3/2h)}{h(h+k)} f_i, \qquad (17\text{-b})$$

$$\tilde{f}'' = \frac{2}{k(h+k)} f_{i+2} - \frac{2}{hk} f_{i+1} + \frac{2}{h(h+k)} f_i. \qquad (17\text{-c})$$

Note that f_i, f_{i+1} and f_{i+2} are not discrete function values at points of the finite-difference grid.

Now consider a centroidal line parallel to the y- grid lines as shown in Figure (7). Using a one-dimensional Taylor series in the y-direction one obtains:

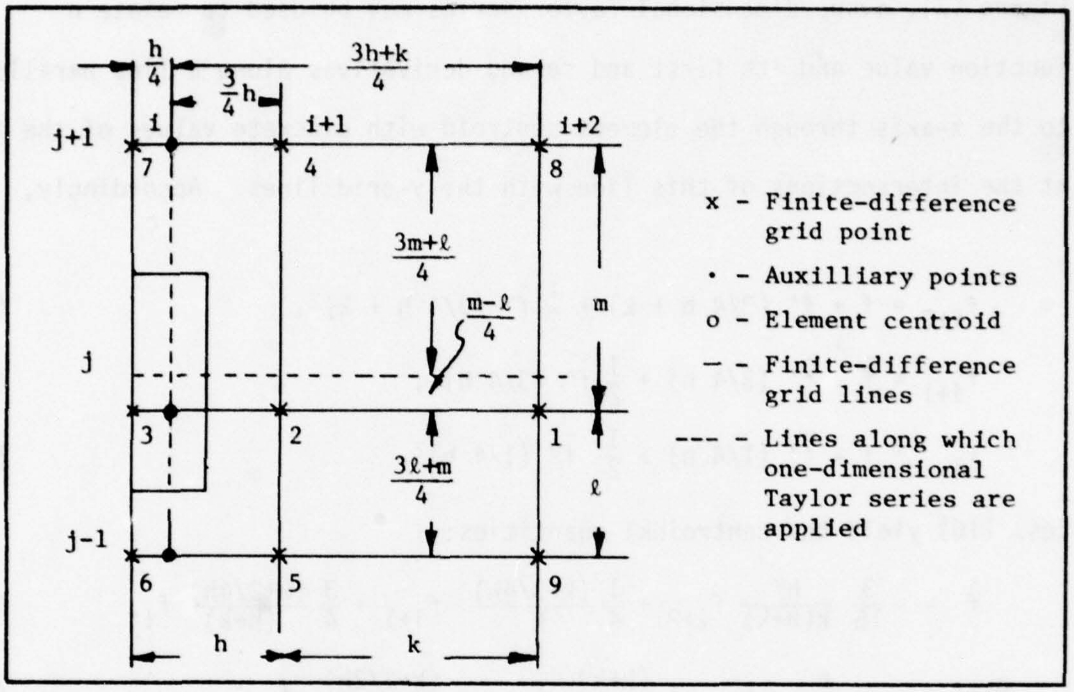


Figure 7. Left exterior boundary element showing lines along which one dimensional function values and derivative values at the element centroid are obtained by Taylor series.

$$f_{j+1} = \tilde{f} + \dot{\tilde{f}}\left(\frac{3m+\ell}{4}\right) + \frac{1}{2}\ddot{\tilde{f}}\left(\frac{3m+\ell}{4}\right)^2, \tag{18-a}$$

$$f_j = \tilde{f} - \dot{\tilde{f}}\left(\frac{m-\ell}{4}\right) + \frac{1}{2}\ddot{\tilde{f}}\left(\frac{m-\ell}{4}\right)^2, \tag{18-b}$$

$$f_{j-1} = \tilde{f} - \dot{\tilde{f}}\left(\frac{3\ell+m}{4}\right) + \frac{1}{2}\ddot{\tilde{f}}\left(\frac{3\ell+m}{4}\right)^2. \tag{18-c}$$

Eqs. (18) yield the centroidal quantities:

$$\tilde{f} = \frac{(m-\ell)(3\ell+m)}{16\, m\, (\ell+m)}\, f_{j+1} + \frac{(3m+\ell)(3\ell+m)}{16\, \ell\, m}\, f_j - \frac{(m-\ell)(3m+\ell)}{16\, \ell\, (\ell+m)}\, f_{j-1}, \tag{19-a}$$

$$\dot{\tilde{f}} = \frac{1}{2m} f_{j+1} + \left(\frac{1}{2\ell} - \frac{1}{2m}\right) f_j - \frac{1}{2\ell} f_{j-1}, \tag{19-b}$$

$$\ddot{\tilde{f}} \quad \frac{2}{m(\ell+m)} f_{j+1} - \frac{2}{\ell m} f_j + \frac{2}{\ell(\ell+m)} f_{j-1}. \tag{19-c}$$

In the foregoing developments $(\ )'$, $(\ )''$ and $(\dot{\ })$, $(\ddot{\ })$, signify differentiation with respect to the x- and y- coordinates, respectively. Note that h, k and ℓ, m are grid spacings in the x- and y- directions, respectively.

Eqs. (17) and (19) can be employed to express a two-dimensional function value and its derivatives as linear combinations of 9 discrete centroidal quantities associated with function values at the grid points. Accordingly, one obtains a transformation of the form:

$$\tilde{f}_i = b_{ij}\, g_j, \quad i = 1,2,\ldots,6; \; j = 1,2,\ldots,9. \tag{20}$$

The vectors $\tilde{f}_i$ and g_j are defined as:

$$[\tilde{f}]^T = [\tilde{f}, \tilde{f}', \dot{\tilde{f}}, \tilde{f}'', \dot{\tilde{f}}', \ddot{\tilde{f}}], \tag{21-a}$$

and

$$[g]^T = [g_1, g_2, g_3, \ldots, g_9]. \tag{22-a}$$

The g_i are discrete values of the function f at the 9 surrounding finite-difference grid points. The transformation matrix b_{ij} is different for each type of exterior and interior boundary element, corner element, or interior element. The procedure used to develop the transformation matrix, b_{ij}, is given in Appendix B.

The centroidal values of the displacements $(\tilde{w}, \tilde{u}, \tilde{v})$ and their first and second order derivatives at the element centroid can be determined via Eq. (20). Accordingly, one can obtain the transformation

$$\tilde{d}_i = c_{ir}\, q_r,\ i=1,2,\ldots,12;\ r=1,2,\ldots,27. \tag{23}$$

The vector q_r is defined as:

$$[q]^T = [W_1, U_1, V_1, W_2, U_2, V_2, \ldots, W_9, U_9, V_9], \tag{24}$$

where W_i, U_i, V_i are displacement components at the nine finite-difference grid points associated with an element. The matrix c_{ir} has the structure shown in Figure (8).

Element Strain Energy. The strain energy of a typical element can now be expressed in terms of its grid-point degrees of freedom. Accordingly, substitution of Equation (23) into Equation (14) yields:

$$2U = A_{rs}\, q_r\, q_s + A_{rst}\, q_r\, q_s\, q_t + A_{rstu}\, q_r\, q_s\, q_t\, q_u, \tag{25}$$

where

$$A_{rs} = a_{ij}\, c_{ir}\, c_{js}, \tag{26-a}$$

$$A_{rst} = a_{ijk}\, c_{ir}\, c_{js}\, c_{kt}, \tag{26-b}$$

$$A_{rstu} = a_{ijk\ell}\, c_{ir}\, c_{js}\, c_{kt}\, c_{\ell u}. \tag{26-c}$$

$$
\left[\begin{array}{ccccccccccccccccccccccccccc}
0 & b_{11} & 0 & 0 & b_{12} & 0 & 0 & b_{13} & 0 & 0 & b_{14} & 0 & 0 & b_{15} & 0 & 0 & b_{16} & 0 & 0 & b_{17} & 0 & 0 & b_{18} & 0 & 0 & b_{19} & 0 \\
0 & b_{21} & 0 & 0 & b_{22} & 0 & 0 & b_{23} & 0 & 0 & b_{24} & 0 & 0 & b_{25} & 0 & 0 & b_{26} & 0 & 0 & b_{27} & 0 & 0 & b_{28} & 0 & 0 & b_{29} & 0 \\
0 & b_{31} & 0 & 0 & b_{32} & 0 & 0 & b_{33} & 0 & 0 & b_{34} & 0 & 0 & b_{35} & 0 & 0 & b_{36} & 0 & 0 & b_{37} & 0 & 0 & b_{38} & 0 & 0 & b_{39} & 0 \\
0 & 0 & b_{11} & 0 & 0 & b_{12} & 0 & 0 & b_{13} & 0 & 0 & b_{14} & 0 & 0 & b_{15} & 0 & 0 & b_{16} & 0 & 0 & b_{17} & 0 & 0 & b_{18} & 0 & 0 & b_{19} \\
0 & 0 & b_{21} & 0 & 0 & b_{22} & 0 & 0 & b_{23} & 0 & 0 & b_{24} & 0 & 0 & b_{25} & 0 & 0 & b_{26} & 0 & 0 & b_{27} & 0 & 0 & b_{28} & 0 & 0 & b_{29} \\
0 & 0 & b_{31} & 0 & 0 & b_{32} & 0 & 0 & b_{33} & 0 & 0 & b_{34} & 0 & 0 & b_{35} & 0 & 0 & b_{36} & 0 & 0 & b_{37} & 0 & 0 & b_{38} & 0 & 0 & b_{39} \\
b_{11} & 0 & 0 & b_{12} & 0 & 0 & b_{13} & 0 & 0 & b_{14} & 0 & 0 & b_{15} & 0 & 0 & b_{16} & 0 & 0 & b_{17} & 0 & 0 & b_{18} & 0 & 0 & b_{19} & 0 & 0 \\
b_{21} & 0 & 0 & b_{22} & 0 & 0 & b_{23} & 0 & 0 & b_{24} & 0 & 0 & b_{25} & 0 & 0 & b_{26} & 0 & 0 & b_{27} & 0 & 0 & b_{28} & 0 & 0 & b_{29} & 0 & 0 \\
b_{31} & 0 & 0 & b_{32} & 0 & 0 & b_{33} & 0 & 0 & b_{34} & 0 & 0 & b_{35} & 0 & 0 & b_{36} & 0 & 0 & b_{37} & 0 & 0 & b_{38} & 0 & 0 & b_{39} & 0 & 0 \\
b_{41} & 0 & 0 & b_{42} & 0 & 0 & b_{43} & 0 & 0 & b_{44} & 0 & 0 & b_{45} & 0 & 0 & b_{46} & 0 & 0 & b_{47} & 0 & 0 & b_{48} & 0 & 0 & b_{49} & 0 & 0 \\
b_{51} & 0 & 0 & b_{52} & 0 & 0 & b_{53} & 0 & 0 & b_{54} & 0 & 0 & b_{55} & 0 & 0 & b_{56} & 0 & 0 & b_{57} & 0 & 0 & b_{58} & 0 & 0 & b_{59} & 0 & 0 \\
b_{61} & 0 & 0 & b_{62} & 0 & 0 & b_{63} & 0 & 0 & b_{64} & 0 & 0 & b_{65} & 0 & 0 & b_{66} & 0 & 0 & b_{67} & 0 & 0 & b_{68} & 0 & 0 & b_{69} & 0 & 0
\end{array}\right]
$$

Figure 8. Structure of Transformation Matrix c_{ij}.

The tensors A_{rs}, A_{rst}, A_{rstu} are completely symmetrical because a_{ij}, a_{ijk}, and $a_{ijk\ell}$ are completely symmetrical. This is an important consideration since the variational procedures proceed more directly because of this symmetry. Details of the development of the components of these tensors are given in Appendix B.

We introduce the second order, symmetrical, tensors $N1_{rs}$ and $N2_{rs}$ through the relations:

$$\frac{1}{2} A_{rst}\, q_t = \frac{1}{6} N1_{rs}, \tag{27}$$

and

$$A_{rstu}\, q_t\, q_u = \frac{1}{6} N2_{rs}. \tag{28}$$

The element strain energy can now be expressed in the preferred form:

$$U = \{\frac{1}{2} A_{rs} + \frac{1}{6} N1_{rs} + \frac{1}{12} N2_{rs}\}\, q_r\, q_s. \tag{29}$$

The tensors $N1_{rs}$ and $N2_{rs}$ depend linearly and quadratically upon the displacement vector q_r. Also it should be observed that they are symmetrical.

Element External Potential Energy. The potential energy associated with the surface forces acting on an element is given by:

$$\Omega = -\tilde{R}_i\, \tilde{d}_i = -\tilde{R}_i\, c_{ij}\, q_j = -R_j\, q_j, \tag{30}$$

where $$R_j = \tilde{R}_i\, c_{ij}. \tag{31}$$

The R_j are generalized external surface forces that are independent of q_j.

System Total Potential Energy. The potential energies of the internal and external forces for a typical element are given by Equations (29) and (30). The total potential energy for a system consisting of N elements is

$$V = \sum_{i=1}^{N} (U_i + \Omega_i), \tag{32}$$

where the summation is to be carried out in the usual manner for assembling a structural stiffness matrix.

From this point forward it is assumed that the total potential energy associated with the complete system has been constructed. It can be written in the form:

$$V = (\frac{1}{2} A_{rs} + \frac{1}{6} N1_{rs} + \frac{1}{12} N2_{rs})\, q_r\, q_s - R_s\, q_s. \tag{33}$$

The tensors A_{rs}, $N1_{rs}$, $N2_{rs}$, R_r, and q_r are system quantities, but the two-dimensional matrices retain the symmetry properties accorded to the corresponding element tensors.

Equilibrium Equations. The principle of total potential energy [16] asserts that the equilibrium configurations of a conservative mechanical system correspond to a stationary value of the total potential energy of the system. Accordingly, when appropriate account is taken of the linear dependence of $N1_{rs}$ and the quadratic dependence of $N2_{rs}$ on the displacement vector, q_r, there is obtained, as a result of setting the first order variation of the total potential energy (V) equal to zero, a set of nonlinear, algebraic equations of the form:

$$(A_{rs} + \frac{1}{2} N1_{rs} + \frac{1}{3} N2_{rs})\, q_s - R_r = 0. \tag{34}$$

The subscripts r, s extend over all degrees of freedom of the system.

Stability Conditions. Bifurcation points on the load-deflection curve of a conservative mechanical system are characterized by a second order variation of the total potential energy that ceases to be positive definite. Since the second order variation as obtained from Equation (34) is

$$\delta^2 V = (A_{rs} + N1_{rs} + N2_{rs})\delta q_r \delta q_s, \tag{35}$$

it follows that the critical load corresponds to the smallest value of load parameter for which the coefficient matrix in Equation (35) vanishes. Accordingly, at a bifurcation point:

$$\mathrm{DET}\ (A_{rs} + N1_{rs} + N2_{rs}) = 0. \tag{36-a}$$

Some structures, especially those that possess initial geometric imperfections, and particularly unbalanced fiber-reinforced composite panels, do not exhibit bifurcation type buckling behaviors. A structure that loses its stability other than by bifurcation usually does so as a limit point buckling behavior. Figures (9) and (10) show these types of buckling behavior [17, 18]. Accordingly, for structures that exhibit limit point buckling behavior, the critical load is characterized by a maximum on the load-deflection curve.

A critical load corresponding to a limit point on a generalized load-deflection curve is determined by generating the load-deflection curve. Numerical determination of bifurcationpoints and limit points for the composite panel is discussed in a subsequent section.

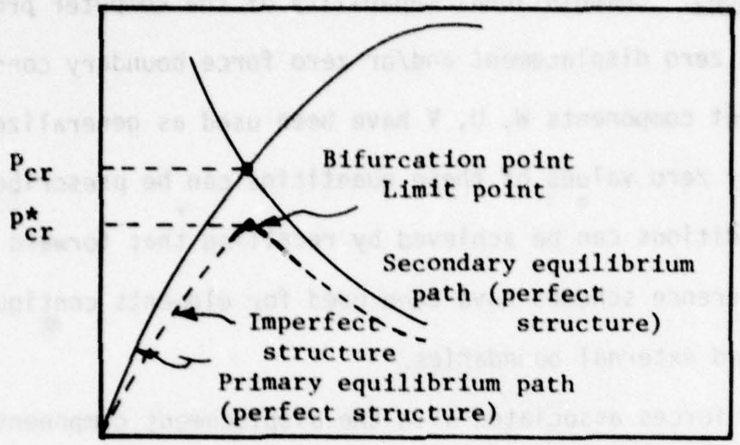


Figure 9. Equilibrium paths depicting bifurcation buckling behavior and limit point buckling behavior typical of imperfection - sensitive structures.

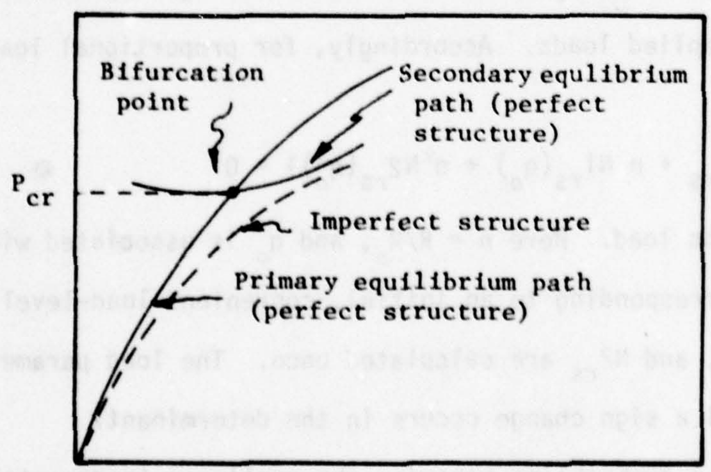


Figure 10. Equilibrium paths depicting bifurcation buckling behavior typical of imperfection-insensitive structures.

Boundary Conditions. Computational capability of the computer program is restricted to zero displacement and/or zero force boundary conditions. Since displacement components W, U, V have been used as generalized coordinated, only zero values of these quantities can be prescribed. Clamped edge conditions can be achieved by recalling that forward and backward finite-difference schemes have been used for elements contiguous with panel internal and external boundaries.

Generalized forces associated with the displacement components W, U, V are automatically specified to be zero if the corresponding displacement component is not specified. Zero edge bending and twisting moments are automatically satisfied because of the variational principle employed when the rotations at the edge are unrestricted.

Numerical Strategies for Bifurcation Analyses. Two fundamental considerations arise when deciding upon a numerical strategy for the detection of bifurcation points: linear prebuckling behavior and nonlinear prebuckling behavior.

For linear prebuckling behavior the prebuckling displacements are linearly related to the applied loads. Accordingly, for proportional loading, one may write:

$$\mathrm{DET}\ \{A_{rs} + n\ N1_{rs}(q_o) + n^2 N2_{rs}(q_o^2)\} = 0 \tag{36-b}$$

at the bifurcation load. Here $n = R/R_o$, and q_o is associated with an equilibrium configuration corresponding to an initial, convenient load-level R_o. The quantities A_{rs}, $N1_{rs}$, and $N2_{rs}$ are calculated once. The load parameter, n, is then incremented until a sign change occurs in the determinant.

For nonlinear prebuckling behavior the nonlinear incremental stiffness matrices, $N1_{rs}$ and $N2_{rs}$, must be recalculated at each load-level. Accordingly, calculation of bifurcation points for nonlinear, prebuckling behavior requires

that:

$$\text{DET}\ \{A_{rs} + N1_{rs}(q) + N2_{rs}(q^2)\} = 0 \tag{36-c}$$

A bifurcation point is bracketed whenever a sign change in the system determinant is detected. Since $N1_{rs}$ and $N2_{rs}$ must be recalculated for each load-level, the detection of a bifurcation point for nonlinear, prebuckling behavior requires the expenditure of considerably more computional time than for linear, prebuckling behavior.

SECTION IV

NUMERICAL METHODS

The Newton-Raphson method or its modified form, appears to have gained a preferred status for use in geometrically nonlinear, structural problems [13]. Accordingly, the modified Newton-Raphson procedure is used in this study. A brief description of the Newton-Raphson method is included here so that a firm grasp of the numerical precedures employed in the computer program can be realized.

Newton-Raphson Procedure. At a given load level, R_r^*, the associated displacement vector, q_r^*, must satisfy Equation (34). Let $\overline{q}_r$ be a reasonably accurate approximation of q_r^* so that:

$$q_r^* = \overline{q}_r + \Delta q_r. \tag{37}$$

Inserting $\overline{q}_r$ into Equation (34) gives:

$$(A_{rs} + \frac{1}{2} N1_{rs} + \frac{1}{3} N2_{rs}) \overline{q}_r - R_r^* = f_r (\overline{q}_i). \tag{38}$$

The vector f_r is a measure of the degree by which the equilibrium at load level R_r^* is not satisfied by the approximate displacement vector $\overline{q}_r$. Pertinent quantities are shown in the one-degree of freedom system of Figure (11).

Expanding f_r in a Taylor series about the point $\overline{q}_r$ gives:

$$f_r(\overline{q}_i + \Delta q_i) = f_r(\overline{q}_i) + \left.\frac{\partial f_r}{\partial q_s}\right|_{\overline{q}_i} \Delta q_s + \text{H.O.T.} \tag{39}$$

The series expansion is truncated after linear terms in Δq_s and $f_r(\overline{q}_i + \Delta q_i)$ is set equal to zero. This yields a set of linear,

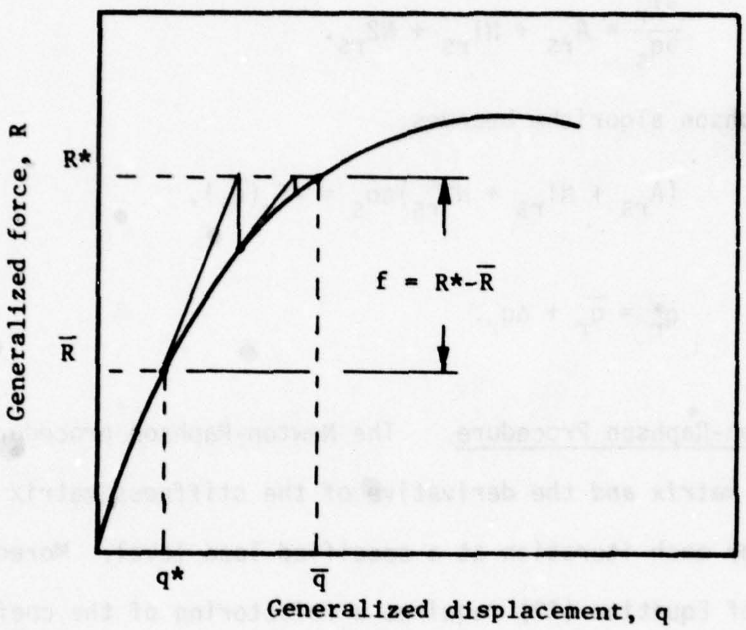


Figure 11. Generalized load - deflection curve depicting pertinent quantities associated with the Newton-Raphson iterative procedure.

algebraic equations for the improvements, Δq_i:

$$\frac{\partial f_r}{\partial q_s}\Big|_{\bar{q}} \Delta q_s = - f_r\Big|_{\bar{q}_i} . \tag{40}$$

The derivatives appearing in Equation (45) can be obtained by differentiating Equation (38). The result is:

$$\frac{\partial f_r}{\partial q_s} = A_{rs} + N1_{rs} + N2_{rs}. \tag{41}$$

The Newton-Raphson algorithm becomes:

$$(A_{rs} + N1_{rs} + N2_{rs})\Delta q_s = -f_r(\bar{q}_i), \tag{42}$$

with

$$q_r^* = \bar{q}_r + \Delta q_r. \tag{43}$$

<u>Modified Newton-Raphson Procedure</u>. The Newton-Raphson procedure requires the stiffness matrix and the derivative of the stiffness matrix to be regenerated for each iteration at a specified load level. Moreover, the solution of Equation (42) requires a refactoring of the coefficient matrix for each iteration. Each of these operations can be costly for large systems of equations. The modified Newton-Raphson procedure maintains the derivative matrice constant for several iterations at a specified load level, thus saving the computer time required to factor the coefficient matrix. Convergence to a desired acuracy using the modified Newton-Raphson procedure is know to require more iterations than the conventional Newton-Raphson procedure. However considerable computer time is saved because the time required to execute the additional iterations required by the modified procedure is generally less than the time required to refactor the coefficient

matrix at each iteration as required by the Newton-Raphson procedure.

Figure 12 shows the pertinent behavior of the modified Newton-Raphson method.

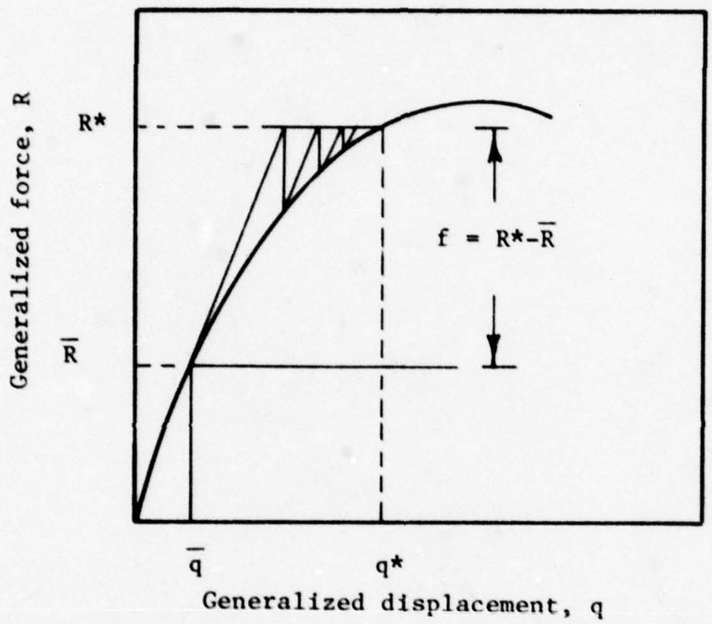


Figure 12. Generalized load-deflection curve depicting pertinent quantities associated with the modified Newton-Raphson iterative method.

SECTION V

NUMERICAL RESULTS

The purpose of this section is to compare some numerical results obtained from the present program with results obtained from STAGS and to demonstrate the types of boundary conditions, applied loads, material properties, and buckling behavior incorporated in the present program.

ISOTROPIC PANEL

Table I contains numerical data obtained via the present program for an isotropic panel whose pertinent material and geometric properties are shown in Figure (13). The plots of these data appear, along with the results obtained from STAGS, as the three lower most curves in Figure (13). Results for a 7 x 7 and for a 13 x 13 finite-difference grid are included to indicate the degree of convergence as the mesh size is decreased. The data from which the curve labeled STAGS was plotted was obtained from Figure (26) of reference [12]. The panel was simply-supported ($W = V = 0$ and $M_x = N_x = 0$) along the curved edges and free ($Q^*_y = M_y = N_y = N_{xy} = 0$) along the free edges. A plot of the system determinant versus load is also shown in the figure and reveals that the collapse load for the panel is about 3 lb (STAGS gives about 2.6 lb).

The upper most curve in Figure (13) represents the load-deflection relationship for the same panel under identical loading conditions when $U = V = W = M_n = 0$ along all edges. The collapse load was determined to be about 8.8 lb when using a 7 x 7 finite-difference grid. No STAGS data were available for comparison. Numerical data applicable to this case is contained in Table III.

Table II contains numerical data obtained from the present program for

TABLE 1

Isotropic panel with a concentrated load applied at the geometric center. The two straight edges are free and the two curved edges are simply supported. (7x7 and 13x13 finite-difference grid).

(7x7 Finite-Difference Grid)

Load (lb)	Determinant	Number of Iterations	Center Deflection (inches)
-0.327	825.9	2	-0.00340
-0.589	802.4	2	-0.00625
-0.850	751.2	3	-0.00923
-1.112	784.5	2	-0.01236
-1.373	783.1	2	-0.01567
-1.635	705.6	3	-0.01922
-1.897	691.6	3	-0.02306
-2.158	675.3	4	-0.02728
-2.420	656.3	5	-0.03205
-2.681	634.2	4	-0.03777
-2.943	584.1	7	-0.04577
-3.205	522.8	*	-0.05849

(13x13 Finite-Difference Grid)

Load (lb)	Determinant	Number of Iterations	Center Deflection (inches)
-0.327	2188.6	4	-0.00345
-0.572	2177.3	2	-0.00617
-0.818	2122.1	2	-0.00904
-1.063	2073.9	2	-0.01210
-1.308	1983.9	3	-0.01538
-1.553	1934.7	3	-0.01894
-1.800	1888.7	3	-0.02285
-2.044	1846.2	4	-0.02725
-2.289	1808.4	4	-0.03232
-2.534	1774.0	4	-0.03847
-2.780	1743.9	4	-0.04667
-3.025	1707.1	6	-0.06142

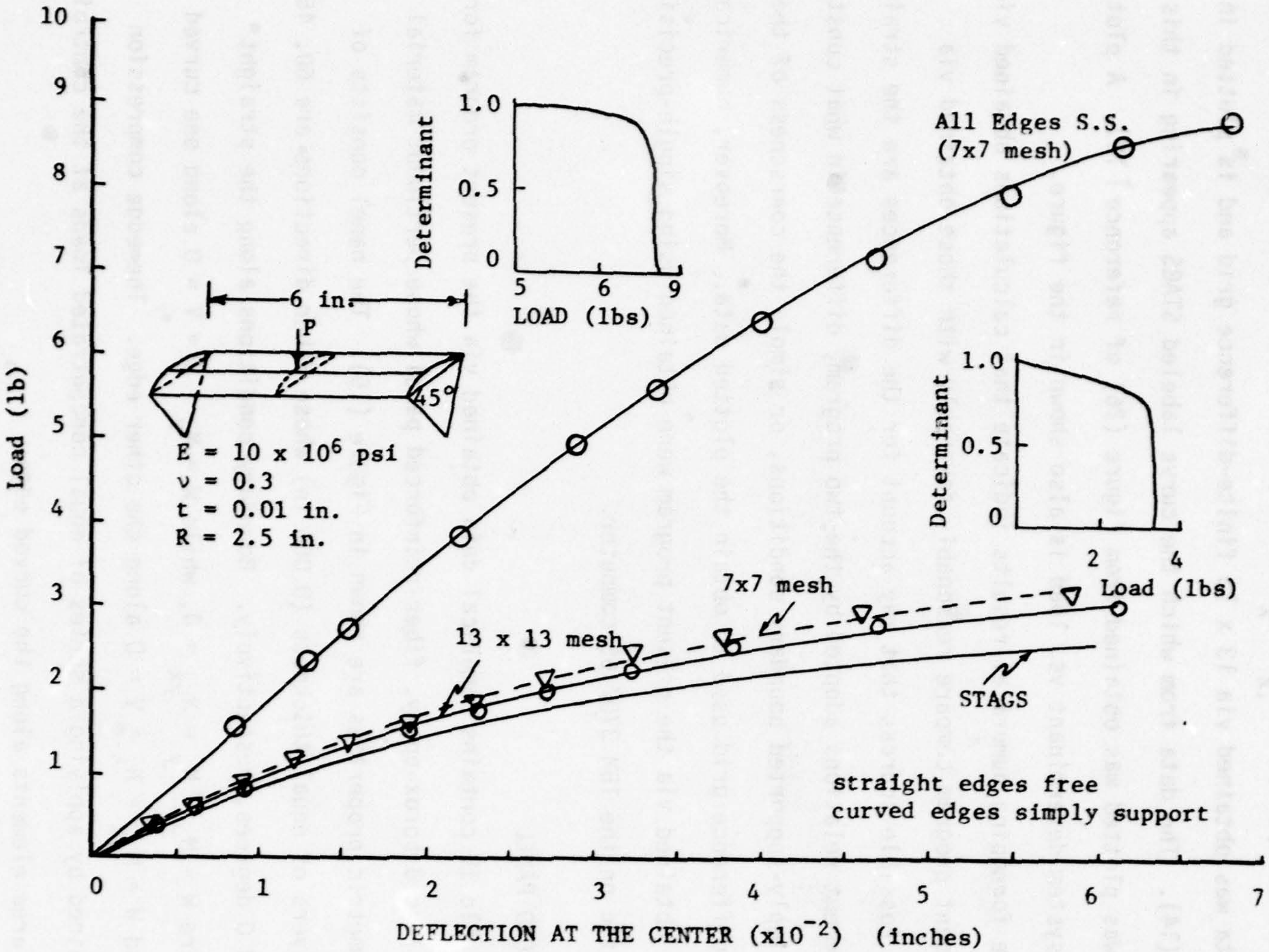


Figure 13. Comparison of present formulation with STAGS for an isotropic panel with straight edges free and curved edges simply-supported and a concentrated load at its geometric center.

the same isotropic panel under identical loading conditions when the curved edges were clamped ($W = W_{,x} = N_x = V = 0$) and the straight edges were free. This data was obtained via 13 x 13 finite-difference grid and is plotted in Figure (14). The data from which the curve labeled STAGS appearing in this figure was plotted was obtained from Figure (26) of reference [12]. A plot of the system determinant vs. load is also shown in the figure.

The foregoing numerical results indicate that calculations obtained via the present program compare reasonably favorably with those obtained via STAGS. Possible sources that may account for the differences are the strain-displacement relations adopted by the two program, difference in what constitutes simply-supported boundary conditions, or simply the coarseness of the finite-difference grid used to obtain the plotted data. Moreover, numerical results obtained via the present program were obtained using single-precision arithmetic on the IBM 370/165 computer.

UNBALANCED PANEL

Table IV contains numerical data obtained via the present program for an unbalanced borox-epoxy, fiber-reinforced panel whose pertinent material and geometric properties are shown in Figure (15). The panel consists of four layers of equal thickness (0.006 in) whose fiber directions are 60, 45, 30, and 0 degrees, respectively. Boundary conditions along the straight edges are $W = M_y = N_y = N_{yx} = 0$, while $W = M_x = U = V = 0$ along one curved edge and $W = M_{yx} = N_x = V = 0$ along the other edge. The edge compression is obtained by applying a series of equal concentrated loads at the centroids of the area elements along the curved edge.

The numerical data of Table IV results in the curved labeled "unbalanced panel" in Figure (15). The buckling behavior is clearly of the limit point type. A plot of the system determinant versus load also shown in this figure.

TABLE 2

Isotropic panel with a concentrated load applied at the geometric center. The two straight edges are free and the two curved edges are clamped. (13x13 Finite-difference grid).

Load (lb)	Determinant	Number of Iterations	Center Deflection (inches)
-1.308	54,258.7	5	-0.01180
-2.126	54,004.1	4	-0.02158
-2.943	53,654.9	4	-0.03582
-3.761	53,011.7	5	-0.06164
-4.251	53,010.8	5	-0.07690
-5.231	53,173.6	6	-0.09570
-6.213	53,553.3	4	-0.10875
-7.194	54,041.6	*	-0.11890

TABLE 3

Isotropic panel with a concentrated load applied at the geometric center. All four edges are simply supported. (7x7 Finite-difference grid).

Load (lb)	Determinant	Number of Iterations	Center Deflection (inches)
-0.040	7029.1	2	-0.000211
-0.157	7024.1	2	-0.000846
-0.589	7012.8	2	-0.00318
-1.570	6983.7	3	-0.00856
-2.374	6958.6	4	-0.01298
-2.708	6911.1	2	-0.01517
-2.855	6902.8	2	-0.01603
-3.002	6895.5	2	-0.01691
-3.188	6884.3	2	-0.01802
-3.414	6871.8	2	-0.01938
-3.826	6845.2	2	-0.02191
-4.326	6808.4	2	-0.02507
-4.915	6758.1	2	-0.02893
-5.592	6684.5	2	-0.03364
-5.886	6647.6	2	-0.03580
-6.409	6558.6	2	-0.03983
-7.194	6358.4	2	-0.04656
-7.979	5874.8	3	-0.05457
-8.240	5478.8	3	-0.05771
-8.502	4272.9	6	-0.06136
-8.764	-5895.3	*	-0.06779

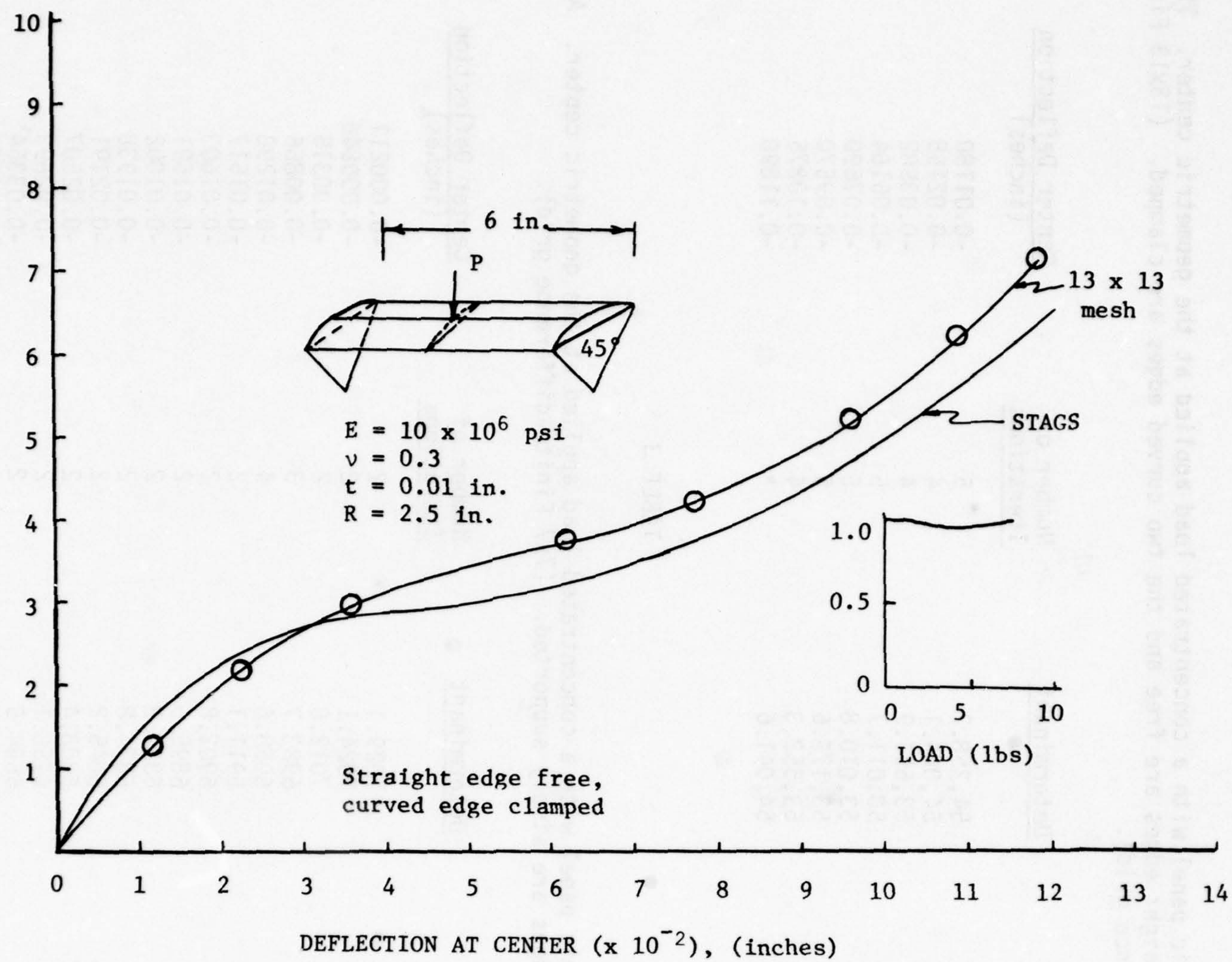


Figure 14. Comparison of present formation with STAGS for an isotropic panel with straight edges free and curved edges clamped and a concentrated load at its geometric center.

BALANCED PANEL

Table V contains numerical data for a panel that differs from the panel discussed in the previous paragraph only in the directions of the fibers of the various layers. The fiber angles ($\pm$ 45 degrees) and stacking sequence have been chosen so as to yield a balanced panel; i.e., a panel for which the coupling between the membrane forces and the moment-resultants does not exist. This data appears in Figure (15) as the curve labeled "balanced panel". A plot of the system determinant versus load also appears in this figure. Apparently, neither bifurcation or limit point buckling has occurred at 540 lb, but apparently the panel is nearing its load carrying capacity as indicated by the load-deflection curve. (The bifurcation load for an isotropic plate with $E = 40 \times 10^6$psi is 548 lb).

The results of a linear, prebuckling, bifurcation analysis is also shown in the insert of Figure (15). A plot of the system determinant vs. load when linear prebuckling behavior is assumed to prevail indicated that bifurcation occurs at about 525 lb.

UNBALANCED PANEL WITH CUT-OUT

Table VI contains numerical data obtained via the present program for the previously discussed panel except that a small rectangular cut-out (0.67 in by 1.0 in) is included at its geometric center. Pertinent material and geometric properties for the panel are shown in Figure (16). Boundary conditions along the straight external edges are $W = M_y = N_y = N_{yx} = 0$, while $W = M_x = U = V = 0$ along one curved, external edge and $W = M_x = N_x = V = 0$ along the other curved, external edge. Along the straight, interior edges $Q^*_y = M_y = U = V = 0$ and along the curved, interior edges $Q^*_y = M_x = U = V = 0$. Edge compression is obtained as described previously.

TABLE 4

Unbalanced boron-epoxy, fiber-reinforced panel under uniform end-compression parallel to the straight edges. All four edges are simply supported with the curved edges subjected to a uniform axial compression.

Load (lb)	Determinant	Number of Iterations	Average Lateral Deflection (inches)
50.0	8660.4	3	0.0108
70.0	8630.2	2	0.0161
90.0	8597.2	2	0.0223
110.0	8557.9	3	0.0296
130.0	8515.5	3	0.0386
150.0	8465.8	3	0.0507
170.0	8395.6	4	0.0696
190.0	-	*	-

TABLE 5

Balanced, boron-epoxy, fiber-reinforced panel under uniform end-compression parallel to the straight edges. All four edges are simply supported with the curved edges subjected to a uniform axial compression.

Load (lb)	Determinant	Number of Iterations	Average Lateral Deflection (inches)
50.0	7696.6	2	0.0062
90.0	7659.9	2	0.0116
130.0	7614.8	2	0.0176
170.0	7563.9	2	0.0243
210.0	7505.3	2	0.0317
250.0	7434.3	3	0.0403
290.0	7352.7	3	0.0503
330.0	7258.5	3	0.0626
370.0	7161.9	3	0.0782
410.0	7078.6	3	0.0987
450.0	7018.6	3	0.1249
490.0	6984.6	3	0.1564
530.0	6964.6	3	0.1924

TABLE 5 (CONT'D)

Linear, prebuckling, befurcation analysis.

Load (lb)	Load Factor, N	Determinant
50.0	1.0	7695.8
75.0	1.5	7673.4
100.0	2.0	7649.1
125.0	2.5	7621.9
150.0	3.0	7592.7
175.0	3.5	7560.9
200.0	4.0	7525.1
225.0	4.5	7484.9
250.0	5.0	7440.1
275.0	5.5	7387.3
300.0	6.0	7326.8
325.0	6.5	7253.5
350.0	7.0	7164.0
375.0	7.5	7051.0
400.0	8.0	6897.7
425.0	8.5	6675.9
450.0	9.0	6306.4
475.0	9.5	5490.4
500.0	10.0	943.9
525.0	10.5	-11,001.6

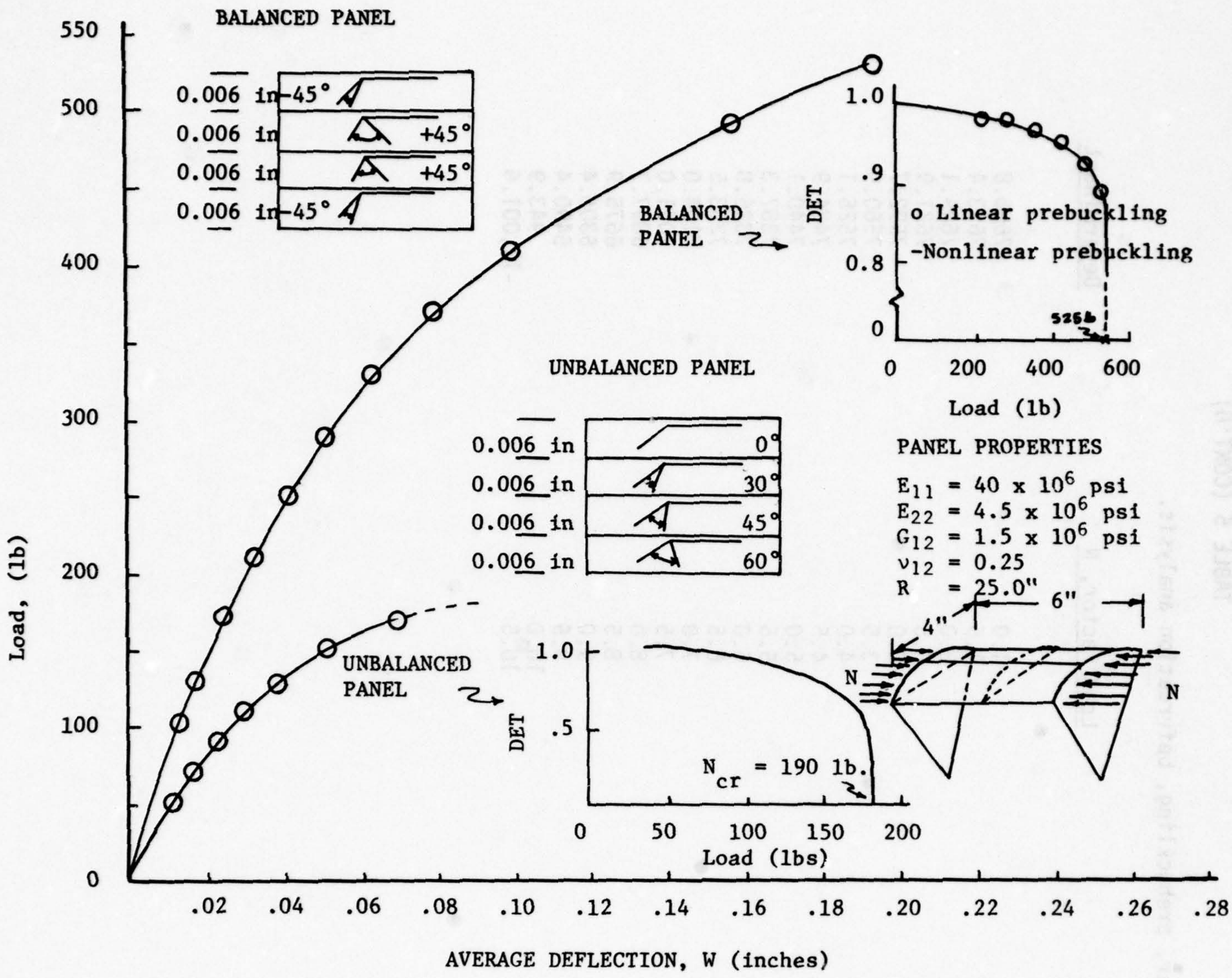


Figure 15. Load-deflection curves for a balanced and an unbalanced, boron-epoxy, panel with simply-supported edges under axial compression.

TABLE 6

Unbalanced boron-epoxy, fiber-reinforced panel under uniform end-compression parallel to the straight edges witha rectangular cut-out at its geometric center. All edges are simply supported.

Load (lb)	Determinant	Number of Iterations	Average Lateral Deflection (inches)
20.0	8264.1	2	0.0043
40.0	8237.6	2	0.0093
60.0	8209.8	2	0.0143
80.0	8179.6	2	0.0203
100.0	8143.9	3	0.0274
110.0	8127.0	2	0.0315
120.0	8107.2	2	0.0360
130.0	8086.3	2	0.0412
140.0	8063.7	2	0.0471
150.0	8034.8	3	0.0542
160.0	8002.8	3	0.0631
170.0	7959.3	3	0.0755
180.0	7849.1	5	0.1025
190.0	*	9	*

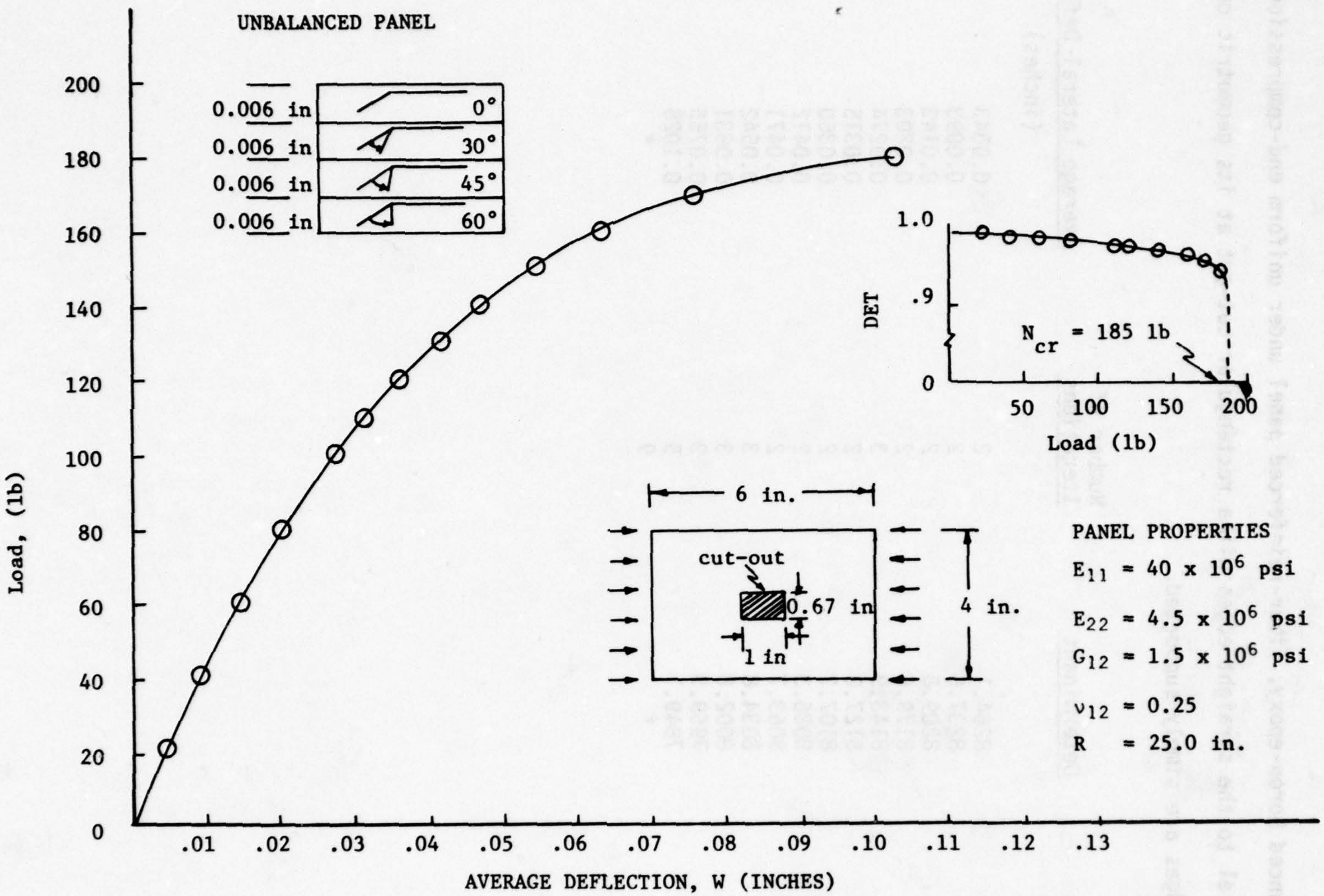


Figure 16. Load-deflection curve for an unbalanced, boron-epoxy, panel with a cut-out at its geometric center. All external edges are simply support and internal edges are free.

TABLE 7

Unbalanced boron-epoxy, fiber-reinforced panel under uniform end-compression with initial geometric imperfection of amplitudes -0.01, -0.03, -0.04, and -0.05. All edges are simply supported.

INITIAL IMPERFECTION AMPLITUDE = -0.01

Load (lb)	Determinant	Number of Iterations	Average Lateral Deflection (inches)
50.0	8660.4	3	0.0108
70.0	8630.2	2	0.0161
90.0	8597.2	2	0.0223
110.0	8557.9	3	0.0296
130.0	8515.5	3	0.0386
150.0	8465.8	3	0.0507
170.0	8395.6	4	0.0696
190.0	*	5	*

INITIAL IMPERFECTION AMPLITUDE = -0.03

Load (lb)	Determinant	Number of Iterations	Average Lateral Deflection (inches)
50.0	8613.7	3	0.0199
70.0	8576.3	3	0.0305
90.0	8533.4	3	0.0436
110.0	8482.3	3	0.0616
130.0	8413.0	4	0.0922
150.0	**	9	**

INITIAL IMPERFECTION AMPLITUDE = -0.04

Load (lb)	Determinant	Number of Iterations	Average Lateral Deflection (inches)
50.0	8442.9	4	0.0654
70.0	8384.8	4	0.1064
90.0	8335.4	4	0.1608
110.0	8312.4	4	0.2234
130.0	8305.0	3	0.2831
150.0	8295.2	4	0.3362
170.0	8267.1	4	0.3832
190.0	8220.6	3	0.4254
210.0	8156.9	3	0.4640
230.0	8077.1	3	0.4997
250.0	7981.9	3	0.5330
270.0	7873.1	3	0.5644

* Did not converge but exhibited a negative determinant after five iterations.

** Did not converge after 9 iterations.

TABLE 7 (CONT'D)

INITIAL IMPERFECTION AMPLITUDE = -0.05

Load (lb)	Determinant	Number of Iterations	Average Lateral Deflection (inches)
50.0	8417.8	4	0.0769
70.0	8373.0	4	0.1194
90.0	8344.3	4	0.1682
110.0	8335.4	4	0.2200
130.0	8332.6	3	0.2705
150.0	8327.0	3	0.3180
170.0	8308.4	3	0.3621
190.0	8274.0	3	0.4030
210.0	8228.4	3	0.4412
230.0	8159.3	3	0.4772
250.0	8083.8	3	0.5113
270.0	7997.8	3	0.5438

INITIAL IMPERFECTION AMPLITUDE = -0.20

Load (lb)	Determinant	Number of Iterations	Average Lateral Deflection (inches)
50.0	7896.2	2	0.0591
70.0	7770.2	2	0.0846
90.0	7778.0	2	0.1100
110.0	7788.2	2	0.1359
130.0	7795.4	2	0.1622
150.0	7799.4	2	0.1888
170.0	7800.0	2	0.2156
190.0	7795.7	2	0.2426
210.0	7786.4	2	0.2698
230.0	7773.1	2	0.2970
250.0	7752.9	2	0.3243
270.0	7727.6	2	0.3515
290.0	7694.0	2	0.3788
310.0	7653.6	2	0.4062

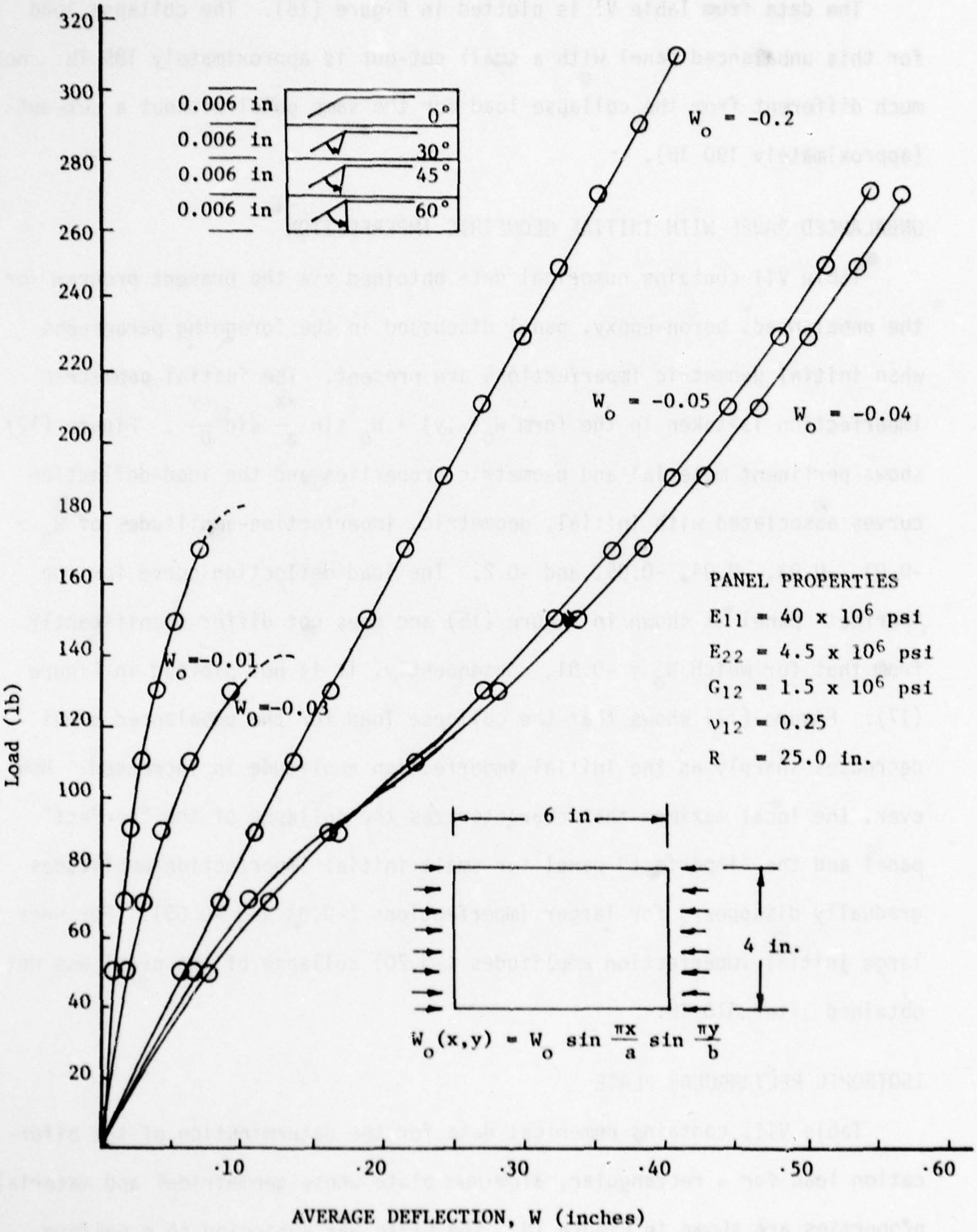


Figure 17. Load-deflection curves for an unbalanced, boron-epoxy, panel with initial geometric imperfections.

The data from Table VI is plotted in Figure (16). The collapse load for this unbalanced panel with a small cut-out is approximately 185 lb: not much different from the collapse load for the same panel without a cut-out (approximately 190 lb).

UNBALANCED PANEL WITH INITIAL GEOMETRIC IMPERFECTION

Table VII contains numerical data obtained via the present program for the unbalanced, boron-epoxy, panel discussed in the foregoing paragraphs when initial geometric imperfections are present. The initial geometric imperfection is taken in the form $W_o(x,y) = W_o \sin \frac{\pi x}{a} \sin \frac{\pi y}{b}$. Figure (17) shows pertinent material and geometric properties and the load-deflection curves associated with initial, geometric, imperfection-amplitudes of W_o = -0.01, -0.03, -0.04, -0.05, and -0.2. The load-deflection curve for the "perfect" panel is shown in Figure (15) and does not differ significantly from that for which W_o = -0.01, consequently, it is not plotted in Figure (17). Figure (17) shows that the collapse load for the unbalanced panel decreases sharply as the initial imperfection amplitude is increased. However, the local maximum that characterizes the collapse of the "perfect" panel and the "imperfect" panel for small initial imperfection-amplitudes gradually disappears for larger imperfections (-0.04 and -0.05). For very large initial imperfection amplitudes (-0.20) collapse of the panel was not obtained after 310 lb.

ISOTROPIC RECTANGULAR PLATE

Table VIII contains numerical data for the determination of the bifurcation load for a rectangular, aluminum plate whose geometrical and material properties are shown in Figure 18. The plate was subjected to a uniform end-compression and all edges were simply-supported. Also shown in Table VIII are numerical data associated with initial geometric imperfections of

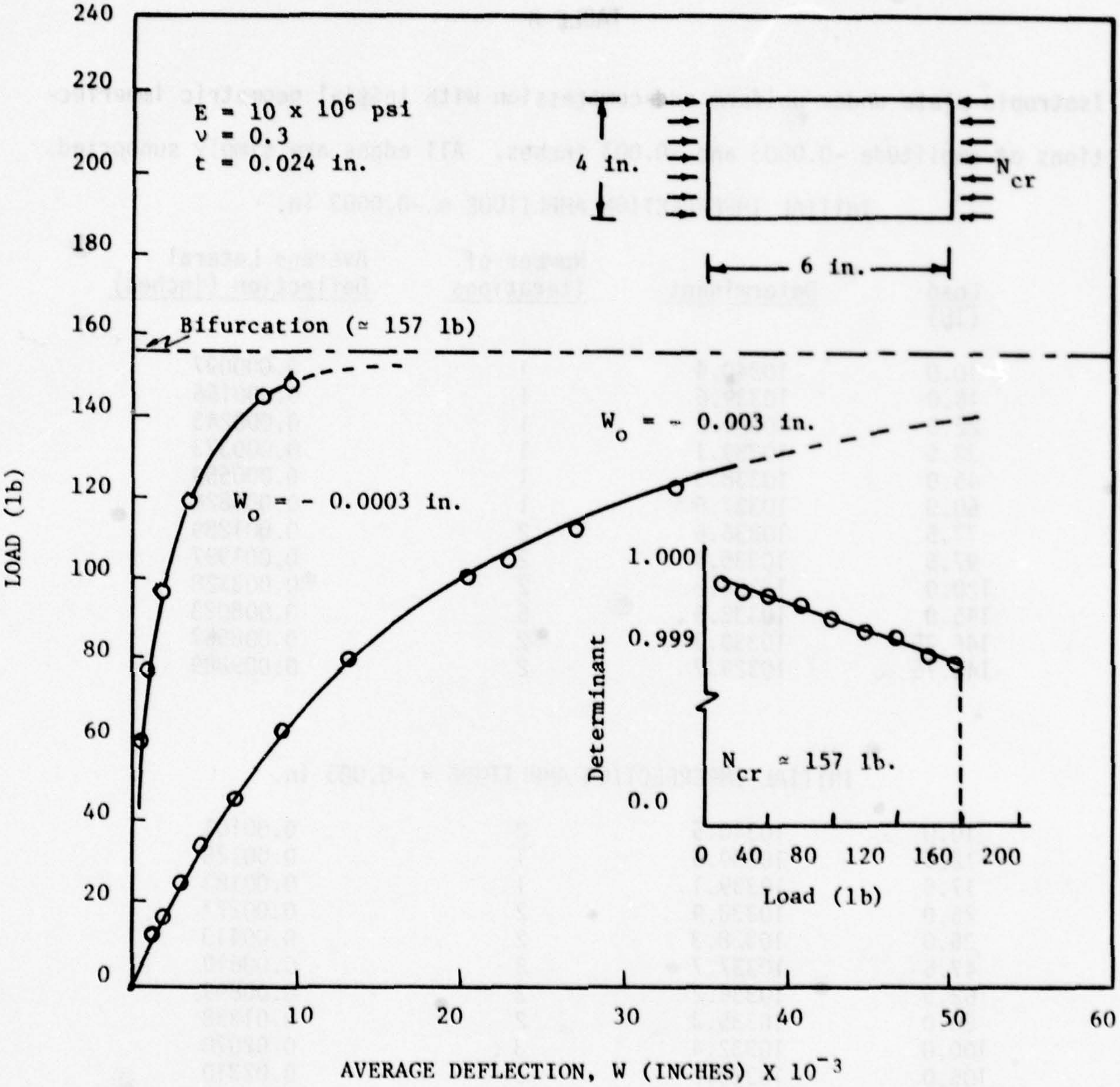


Figure 18. Load-deflection curves for a balanced, isotropic, plate with initial geometric imperfections.

TABLE 8

Isotropic plate under uniform end-compression with initial geometric imperfections of amplitude -0.0003 and -0.003 inches. All edges are simply supported.

INITIAL IMPERFECTION AMPLITUDE = -0.0003 in.

Load (lb)	Determinant	Number of Iterations	Average Lateral Deflection (inches)
10.0	10340.4	1	0.000097
15.0	10339.6	1	0.000156
22.5	10339.2	1	0.000243
32.5	10339.1	1	0.000373
45.0	10338.3	1	0.000559
60.0	10337.6	1	0.000828
77.5	10336.6	2	0.001289
97.5	10335.4	2	0.001997
120.0	10334.2	2	0.003328
145.0	10332.6	5	0.008023
146.25	10330.3	2	0.008557
148.75	10329.7	2	0.009889

INITIAL IMPERFECTION AMPLITUDE = -0.003 in.

Load (lb)	Determinant	Number of Iterations	Average Lateral Deflection (inches)
10.0	10340.5	2	0.00101
12.5	10339.6	1	0.00128
17.5	10339.1	1	0.00183
25.0	10338.9	2	0.00277
35.0	10338.3	2	0.00413
47.5	10337.7	2	0.00610
62.5	10336.2	2	0.00899
80.0	10335.3	2	0.01338
100.0	10332.4	3	0.02070
105.0	10328.8	2	0.02310
112.5	10327.7	2	0.02730
122.5	10325.0	1	0.03340

TABLE 8 (CONT'D)

LINEAR PREBUCKLING BIFURCATION ANALYSIS

Load (lb)	Load Factor, N	Determinant
10.0	1.0	10338.4
15.0	1.5	10337.8
20.0	2.0	10337.7
25.0	2.5	10337.5
30.0	3.0	10337.2
35.0	3.5	10336.9
40.0	4.0	10336.6
45.0	4.5	10336.2
50.0	5.0	10335.9
55.0	5.5	10335.8
60.0	6.0	10335.6
65.0	6.5	10334.9
70.0	7.0	10334.6
75.0	7.5	10334.4
80.0	8.0	10334.0
85.0	8.5	10333.6
95.0	9.5	10332.8
100.0	10.0	10332.6
105.0	10.5	10332.4
110.0	11.0	10332.1
115.0	11.5	10331.7
120.0	12.0	10331.3
125.0	12.5	10331.1
130.0	13.0	10330.7
135.0	13.5	10330.0
140.0	14.0	10329.5
145.0	14.5	10329.5
150.0	15.0	10329.0
155.0	15.5	10328.9
160.0	16.0	-10328.7

amplitudes -0.003 in. and -0.0003 in. These data are plotted in Figure 18 for a 13 x 13 finite-difference grid.

The bifurcation branch of the present program predicts a critical load of magnitude 157 lb. while Timoshenko theory predicts a critical load of 148 lb.(6% high). The load-deflection curves representing imperfection amplitudes of -0.003 in. and -0.0003 in. appear to be approaching the calculated bifurcation load asymptotically.

The foregoing examples demonstrate the types of boundary conditions, applied loads, material properties, and initial imperfections for which the present computer program can render bifurcation and collapse analyses.

As much of the analysis is built into the computer program as seems computationally feasible. Nevertheless, the analyst must make certain decisions regarding an appropriate initial load level, initial load-increment, and convergence criterion for the modified Newton/Raphson procedure. The present program sets this converge criterion at 10^{-4}, but it can be easily altered. Usually, one will not know a priori the order of magnitude of the bifurcation or collapse load for a particular panel; consequently, it is difficult to make wise decisions regarding the initial load-level and initial load-increment. It therefore seems computationally efficient to obtain an approximate buckling load by the linear, prebuckling, eigenvalue routine provided as an option in the present computer program prior to making a complete nonlinear analysis. Moreover, because the severity of the nonlinear effects of the load-deflection curve can not always be ascertained a priori, it may be effective to generate the load-deflection curve in separate segments.

Finally, the present program does not possess the capacity to generate the load-deflection curve beyond a local maximum; i.e., the present coding does not permit calculation of the post-buckled equilibrium configurations. However, the program can be modified to permit calculation of the post-buckled configurations.

APPENDIX A

FORMULAS FOR COEFFICIENTS ASSOCIATED WITH QUADRATIC, CUBIC, AND QUARTIC ELEMENT ENERGY TERMS

Coefficients a_{ij}, a_{ijk}, and a_{ijkl} associated with the quadratic, cubic, and quartic portions of the strain energy of an element are developed in this appendix.

The strain energy of an element is given by the formula:

$$2U = \tilde{N}_{ij}\tilde{\varepsilon}_i\tilde{\varepsilon}_j, \qquad i,j = 1, 2, \ldots, 6. \tag{A-1}$$

$\tilde{N}_{ij}$ is the material stiffness matrix as expressed by Eq. (11-b) and $\tilde{\varepsilon}_i$ are the strain components. The linear and nonlinear portions of $\tilde{\varepsilon}_i$ are denoted by L_i and NL_i, respectively, so that,

$$\tilde{\varepsilon}_i = L_i + NL_i. \tag{A-2}$$

For convenience the tilda over any quantity will be suppressed from this point with the understanding that the strain components and the material stiffness matrix are associated with the centroid of the element area.

Explicit expressions for the linear and nonlinear parts of the strain components are:

$$\begin{aligned} L_1 &= U_{,x} + W_{o,x}W_{,x} \quad , \\ L_2 &= V_{,y} + W/R - W_{o,y}(V/R - W_{,y}) \quad , \\ L_3 &= V_{,x} + U_{,y} - W_{o,x}(V/R - W_{,y}) + W_{o,y}W_{,x} \quad , \end{aligned} \tag{A-3}$$

$$L_4 = -W_{,xx} \quad ,$$

$$L_5 = -W_{,yy} + V_{,y}/R \quad , \tag{A-3}$$

$$L_6 = -2W_{,xy} + \frac{3}{2R} V_{,x} - \frac{1}{2R} U_{,y} \text{ , and}$$

$$NL_1 = \frac{1}{2} W_{,x}^{\,2} + \frac{1}{8} V_{,x}^{\,2} - \frac{1}{4} V_{,x} U_{,y} + \frac{1}{8} U_{,y}^{\,2}$$

$$NL_2 = \frac{1}{2} W_{,y}^{\,2} - \frac{1}{R} VW_{,y} + \frac{1}{2R^2} V^2 - \frac{1}{8} V_{,x}^{\,2} + \frac{1}{4} U_{,y} V_{,x} - \frac{1}{8} U_{,y}^{\,2} \quad ,$$

$$NL_3 = W_{,x} W_{,y} - \frac{1}{R} VW_{,x} \quad , \tag{A-4}$$

$$NL_4 = 0 \quad ,$$

$$NL_5 = 0 \quad ,$$

$$NL_6 = 0 \quad .$$

The energy of an element can be segregated into its quadratic, cubic, and quartic parts using Eqs. (A-1) and (A-2). Thus,

$$2U = N_{ij} L_i L_j + N_{ij} (L_i NL_i + L_i NL_i) + N_{ij} NL_i NL_j \quad , \tag{A-5}$$

where the first, second, and third terms on the right-hand side of Eq. (A-5) are the quadratic, cubic, and quartic contributions to the strain energy of an element. Each of these quantities can be expressed explicitly so that formulas for calculating the coefficients a_{ij}, a_{ijk}, and a_{ijkl} can be determined. These coefficients depend only upon the material and geometric properties of the panel and not upon the selection of a finite-difference grid.

Quadratic Contribution. The following terms result from an expansion of the quadratic portion of the strain energy. Because of symmetry only the unique terms in the expansion need to be formulated.

$i = 1 \quad , \quad j = 1, 2, \ldots, 6:$

$$N_{11}L_1L_1 = N_{11}U^2_{,x} + N_{11}(2W_{o,x}U_{,x}W_{,x} + W^2_{o,x}W^2_{,x}) \quad ,$$

$$(N_{12} + N_{21})L_1L_2 = 2N_{12}(U_{,x}V_{,y} + \frac{1}{R}U_{,x}W)$$

$$+ 2N_{12}(-W_{o,y}U_{,x}V/R + W_{o,y}U_{,x}W_{,y} + W_{o,x}W_{,x}V_{,y}$$

$$+ W_{o,x}W_{,x}W/R - W_{o,x}W_{o,y}W_{,x}V/R$$

$$+ W_{o,x}W_{o,y}W_{,x}W_{,y}) \quad ,$$

$$(N_{13} + N_{31})L_1L_3 = 2N_{13}(U_{,x}V_{,x} + U_{,x}U_{,y})$$

$$+ 2N_{13}(W_{o,x}U_{,x}W_{,y} + W_{o,y}U_{,x}W_{,x} - W_{o,x}U_{,x}V/R$$

$$+ W_{o,x}W_{,x}V_{,x} + W_{o,x}U_{,y}W_{,x} + W^2_{o,x}W_{,x}W_{,y}$$

$$+ W_{o,x}W_{o,y}W^2_{,x} - W^2_{o,x}VW_{,x}/R) \quad ,$$

$$(N_{14} + N_{41})L_1L_4 = 2N_{14}(-U_{,x}W_{,xx}) - 2N_{14}(W_{o,x}W_{,x}W_{,xx}) \quad ,$$

$$(N_{15} + N_{51})L_1L_5 = 2N_{15}(-U_{,x}W_{,yy} + U_{,x}V_{,y}/R) + 2N_{15}(-W_{o,x}W_{,x}W_{,yy}$$

$$+ W_{o,x}V_{,y}W_{,x}) \quad ,$$

$$(N_{16} + N_{61})L_1L_6 = 2N_{16}(-2U_{,x}W_{,xy} + \frac{3}{2R}U_{,x}V_{,x} - \frac{1}{2R}U_{,x}U_{,y})$$
$$+ 2N_{16}(-2W_{o,x}W_{,x}W_{,xy} + \frac{3}{2R}W_{o,x}V_{,x}W_{,x}$$
$$- \frac{1}{2R}W_{o,x}U_{,y}W_{,x}) \quad ;$$

$i = 2 \quad , j = 2, 3, \ldots, 6 \quad :$

$$N_{22}L_2L_2 = N_{22}(V^2_{,y} + 2V_{,y}W/R + W^2/R^2)$$
$$+ N_{22}(W^2_{o,y}V^2/R^2 + W^2_{o,y}W^2_{,y} - \frac{2}{R}W_{o,y}VV_{,y} + 2W_{o,y}V_{,y}W_{,y}$$
$$- \frac{2}{R^2}W_{o,y}VW + \frac{2}{R}W_{o,y}WW_{,y} - \frac{2}{R}W^2_{o,y}VW_{,y}) \quad ,$$

$$(N_{23} + N_{32})L_2L_3 = 2N_{23}(V_{,x}V_{,y} + WV_{,x}/R + U_{,y}V_{,y} + WU_{,y}/R)$$
$$+ 2N_{23}(W_{o,x}V_{,y}W_{,y} + W_{o,y}V_{,y}W_{,x} - \frac{1}{R}W_{o,x}VV_{,y} + \frac{1}{R}W_{o,x}WW_{,y}$$
$$+ \frac{1}{R}W_{o,y}WW_{,x} - \frac{1}{R^2}W_{o,x}VW - \frac{1}{R}W_{o,y}W_{o,x}VW_{,y}$$
$$- \frac{1}{R}W_{o,y}VV_{,x} - \frac{1}{R}W_{o,y}U_{,y}V - \frac{1}{R}W^2_{o,y}VW_{,x} + \frac{1}{R^2}W_{o,x}W_{o,y}V^2$$
$$+ W_{o,y}V_{,x}W_{,y} + W_{o,y}U_{,y}W_{,y} + W_{o,x}W_{o,y}W^2_{,y}$$
$$+ W^2_{o,y}W_{,x}W_{,y} - \frac{1}{R}W_{o,x}W_{o,y}VW_{,y}) \quad ,$$

$$(N_{24} + N_{42})L_2L_4 = 2N_{24}(-V_{,y}W_{,xx} - \frac{1}{R}WW_{,xx})$$
$$+ 2N_{24}(W_{o,y}VW_{,xx}/R - W_{o,y}W_{,y}W_{,xx}) \quad ,$$

$$(N_{25} + N_{52})L_2L_5 = 2N_{25}(-V_{,y}W_{,yy} - WW_{,yy}/R + V^2_{,y}/R + WV_{,y}/R^2)$$
$$+ 2N_{25}(W_{o,y}VW_{,yy}/R - W_{o,y}VV_{,y}/R^2 - W_{o,y}W_{,y}W_{,yy}$$
$$+ W_{o,y}V_{,y}W_{,y}/R) \quad ,$$

$$(N_{26} + N_{62})L_2L_6 = 2N_{26}(-2V_{,y}W_{,xy} + \frac{3}{2R} V_{,x}V_{,y} - \frac{1}{2R} U_{,y}V_{,y}$$

$$- \frac{2}{R} WW_{,xy} + \frac{3}{2R^2} WV_{,x} - \frac{1}{2R^2} WU_{,y})$$

$$+ 2N_{26}(\frac{2}{R} W_{o,y}VW_{,xy} - \frac{3}{2R^2} W_{o,y}VV_{,x} + \frac{1}{2R^2} W_{o,y}VU_{,y}$$

$$-2W_{o,y}W_{,y}W_{,xy} + \frac{3}{2R} W_{o,y}V_{,x}W_{,y}$$

$$- \frac{1}{2R} W_{o,y}U_{,y}W_{,y}) \quad ;$$

$i = 3 \quad , \quad j = 3, 4, 5, 6 \quad :$

$$N_{33}L_3L_3 = N_{33}(V^2_{,x} + 2V_{,x}U_{,y} + U^2_{,y})$$

$$+ N_{33}(W^2_{o,x}W^{\,2}_{,y} + W^2_{o,y}W^2_{,x} + \frac{1}{R^2} W^2_{o,x}V^2 + 2W_{o,x}V_{,x}W_{,y}$$

$$+ 2W_{o,x}U_{,y}W_{,y} + 2W_{o,x}W_{o,y}W_{,x}W_{,y} - \frac{2}{R} W^2_{o,x}VW_{,y}$$

$$+ 2W_{o,y}V_{,x}W_{,x} + 2W_{o,y}U_{,y}W_{,x} - \frac{2}{R} W_{o,x}W_{o,y}VW_{,x}$$

$$- \frac{2}{R} W_{o,x}VV_{,x} - \frac{2}{R} W_{o,x}VU_{,y}) \quad ,$$

$$(N_{34} + N_{43})L_3L_4 = 2N_{34}(-V_{,x}W_{,xx} - U_{,y}W_{,xx})$$

$$+ 2N_{34}(-W_{o,x}W_{,y}W_{,xx} - W_{o,y}W_{,x}W_{o,xx} + \frac{1}{R} W_{o,x}VW_{,xx}) \quad ,$$

$$(N_{35} + N_{53})L_3L_5 = 2N_{35}(-V_{,x}W_{,yy} - U_{,y}W_{,yy} + \frac{1}{R} V_{,x}V_{,y} + \frac{1}{R} U_{,y}V_{,y})$$

$$+ 2N_{35}(-W_{o,x}W_{,y}W_{,yy} - W_{o,y}W_{,x}W_{,yy} + \frac{1}{R} W_{o,x}VW_{,yy}$$

$$+ \frac{1}{R} W_{o,x}V_{,y}W_{,y} + \frac{1}{R} W_{o,y}V_{,y}W_{,x} - \frac{1}{R^2} W_{o,x}VV_{,y})$$

0	0	0	0	0	0	0	0	0	0	0	0
	N_{11}	$N_{13} - \frac{N_{16}}{2R}$	0	$N_{13} + \frac{3N_{16}}{2R}$	$N_{12} + \frac{N_{15}}{R}$	$\frac{N_{12}}{R}$	0	0	$-N_{14}$	$-2N_{16}$	$-N_{15}$
		$N_{33} - \frac{N_{36}}{2R} + \frac{N_{66}}{4R^2}$	0	$N_{33} + \frac{N_{36}}{R} - \frac{3N_{66}}{4R^2}$	$N_{23} - \frac{N_{26}}{2R} + \frac{N_{35}}{R} - \frac{N_{56}}{2R^2}$	$\frac{N_{23}}{R} - \frac{N_{26}}{2R^2}$	0	0	$-N_{34} + \frac{N_{46}}{2R}$	$-2N_{36} + \frac{N_{66}}{R}$	$-N_{35} + \frac{N_{56}}{2R}$
			0	0	0	0	0	0	0	0	0
				$N_{33} + \frac{9N_{66}}{4R^2} + \frac{3N_{36}}{2R}$	$N_{23} + \frac{3}{2R} N_{26} + \frac{N_{35}}{R} + \frac{3N_{56}}{2R^2}$	$\frac{N_{23}}{R} + \frac{3N_{26}}{2R^2}$	0	0	$-N_{34} - \frac{3N_{46}}{2R}$	$-2N_{36} - \frac{3N_{66}}{R}$	$-N_{35} - \frac{3N_{56}}{2R}$
		SYM			$N_{22} + \frac{2N_{25}}{R} + \frac{N_{55}}{R^2}$	$\frac{N_{22}}{R} + \frac{N_{25}}{R^2}$	0	0	$-N_{24} - \frac{N_{45}}{R}$	$-2N_{26} - \frac{2N_{56}}{R}$	$-N_{25} - \frac{N_{55}}{R}$
						$\frac{N_{22}}{R^2}$	0	0	$-\frac{N_{24}}{R}$	$-\frac{2N_{26}}{R}$	$-\frac{N_{25}}{R}$
							0	0	0	0	0
								0	0	0	0
									N_{44}	$2N_{46}$	N_{45}
										$4N_{66}$	$2N_{56}$
											N_{55}

Figure A-1. Linear Stiffness Matrix for Fiber-Reinforced Panel.

The following formulas give the contribution of the initial, lateral, geometric imperfection to the quadratic portion of the strain energy. Note that these contributions form a symmetric matrix so that only nonzero and upper triangular elements are listed.

2,4 : $-\frac{1}{R}(N_{12} W_{o,y} + N_{13} W_{o,x})$

2,8 : $N_{11} W_{o,x} + N_{13} W_{o,y}$

2,9 : $N_{12} W_{o,y} + N_{13} W_{o,x}$

3,4 : $(-N_{33}/R + N_{36}/2R^2) W_{o,x} + (-N_{23}/R + N_{26}/2R^2) W_{o,y}$

3,8 : $(N_{33} - N_{36}/2R) W_{o,y} + (-N_{16}/2R + N_{13}) W_{o,x}$

3,9 : $(N_{33} - N_{36}/2R) W_{o,x} + (-N_{26}/2R + N_{23}) W_{o,y}$

4,4 : $\frac{1}{R^2}(N_{33} W_o^2{}_{,x} + 2N_{23} W_{o,x} W_{o,y} + N_{22} W_o^2{}_{,y})$

4,5 : $-(N_{33}/R + \frac{3N_{36}}{2R^2}) W_{o,x} - (\frac{N_{23}}{R} + \frac{3N_{26}}{2R^2}) W_{o,y}$

4,6 : $-(\frac{N_{35}}{R^2} + \frac{N_{23}}{R}) W_{o,x} - (\frac{N_{22}}{R} + \frac{N_{25}}{R^2}) W_{o,y}$

4,7 : $-\frac{1}{R^2}(N_{22} W_{o,y} + N_{23} W_{o,x})$

4,8 : $-[\frac{N_{13}}{R} W_{o,x}^2 + (\frac{N_{33}}{R} + \frac{N_{12}}{R}) W_{o,x} W_{o,y} + \frac{N_{23}}{R} W_o^2{}_{,y}]$

4,9 : $-\frac{1}{R}(N_{33} W_o^2{}_{,x} + 2N_{23} W_{o,x} W_{o,y} + N_{22} W_o^2{}_{,y})$

4,10 : $\frac{1}{R}(N_{34} W_{o,x} + N_{24} W_{o,y})$

4,11 : $\frac{2}{R}(N_{36} W_{o,x} + N_{26} W_{o,y})$

4,12 : $\frac{1}{R}(N_{35} W_{o,x} + N_{25} W_{o,y})$

5,8 : $(N_{13} + \frac{3N_{16}}{2R}) W_{o,x} + (N_{33} + \frac{3N_{36}}{2R}) W_{o,y}$

5,9 : $(N_{33} + \frac{3N_{36}}{2R}) W_{o,x} + (N_{23} + \frac{3N_{26}}{2R}) W_{o,y}$

6,8 : $(N_{12} + \frac{N_{15}}{R}) W_{o,x} + (N_{23} + \frac{N_{35}}{R}) W_{o,y}$

6,9 : $(N_{23} + \frac{N_{35}}{R})\, W_{o,x} + (N_{22} + \frac{N_{25}}{R})\, W_{o,y}$

7,8 : $\frac{1}{R}\,(N_{12}\, W_{o,x} + N_{23}\, W_{o,y})$

7,9 : $\frac{1}{R}\,(N_{23}\, W_{o,x} + N_{22}\, W_{o,y})$

8,8 : $N_{11}\, W_o^2{}_{,x} + 2N_{13}\, W_{o,x}\, W_{o,y} + N_{33}\, W_o^2{}_{,y}$

8,9 : $N_{13}\, W_o^2{}_{,x} + (N_{12} + N_{33})\, W_{o,x}\, W_{o,y} + N_{23}\, W_o^2{}_{,y}$

8,10 : $-N_{14}\, W_{o,x} - N_{34}\, W_{o,y}$

8,11 : $-2N_{16}\, W_{o,x} - 2N_{36}\, W_{o,y}$

8,12 : $-N_{15}\, W_{o,x} - N_{35}\, W_{o,y}$

9,9 : $N_{33}\, W_o^2{}_{,x} + 2N_{23}\, W_{o,x}\, W_{o,y} + N_{22}\, W_o^2{}_{,y}$

9,10 : $-N_{34}\, W_{o,x} - N_{24}\, W_{o,y}$

9,11 : $-2N_{36}\, W_{o,x} - 2N_{26}\, W_{o,y}$

9,12 : $-N_{35}\, W_{o,x} - N_{25}\, W_{o,y}$

$$(N_{36} + N_{63})L_3L_6 = 2N_{36}(-2V_{,x}W_{,xy} + \frac{3}{2R}V^2_{,x} - \frac{1}{2R}V_{,x}U_{,y} - 2U_{,y}W_{,xy}$$

$$+ \frac{3}{2R}U_{,y}V_{,x} - \frac{1}{2R}U^2_{,y})$$

$$+ 2N_{36}(-2W_{o,x}W_{,y}W_{,xy} - 2W_{o,y}W_{,x}W_{,xy} + \frac{2}{R}W_{o,x}VW_{,xy}$$

$$+ \frac{3}{2R}W_{o,x}V_{,x}W_{,xy} + \frac{3}{2R}W_{o,y}V_{,x}W_{,x} - \frac{3}{2R}W_{o,x}VV_{,x}$$

$$- \frac{1}{2R}W_{o,x}U_{,y}W_{,y} - \frac{1}{2R}W_{o,y}U_{,y}W_{,x} + \frac{1}{2R^2}W_{o,x}VU_{,y}) \quad ;$$

$i = 4 \quad , \quad j = 4, 5, 6$:

$$N_{44}L_4L_4 = N_{44}W^2_{,xx} \quad ,$$

$$(N_{45} + N_{54})L_4L_5 = 2N_{45}(W_{,xx}W_{,yy} - \frac{1}{R}W_{,xy}V_{,y}) \quad ,$$

$$(N_{46} + N_{64})L_4L_6 = 2N_{46}(2W_{,xx}W_{,yy} - \frac{3}{2R}V_{,x}W_{,xx} + \frac{1}{2R}U_{,y}W_{,xx}) \quad ;$$

$i = 5 \quad , \quad j = 5, 6$:

$$N_{55}L_5L_5 = N_{55}(W^2_{,yy} - 2/RW_{,yy}V_{,y} + \frac{1}{R^2}V^2_{,y}) \quad ,$$

$$(N_{56} + N_{65})L_5L_6 = 2N_{56}(2W_{,xy}W_{,yy} - \frac{3}{2R}V_{,x}W_{,yy} + \frac{1}{2R}U_{,y}W_{,yy}$$

$$- \frac{2}{R}V_{,y}W_{,xy} + \frac{3}{2R^2}V_{,x}V_{,y} - \frac{1}{2R^2}U_{,y}V_{,y}) \quad ;$$

$i = 6 \quad , \quad j = 6:$

$$N_{66}L_6L_6 = N_{66}(4W^2_{,xy} + \frac{9}{4R^2} V^2_{,x} + \frac{1}{4R^2} U^2_{,y} - \frac{6}{R} V_{,x}W_{,xy}$$

$$- \frac{3}{2R^2} U_{,y}V_{,x} + \frac{2}{R} U_{,y}W_{,xy}) \; .$$

The coefficients appearing in the foregoing expressions are collected and assembled in two 12x12 arrays in accordance with vector $[\tilde{d}]$ of Eq. (13). These arrays are shown in Figures (A-1) and (A-2) and represent, respectively, the quadratic strain energy contribution in the absence of initial geometric imperfections and the additional contributions that arise out of the presence of imperfections.

Cubic Contributions. The following terms result from an expansion of the cubic portion of the strain energy of an element.

$i = 1 \quad , \quad j = 1, 2, \ldots, 6:$

$$N_{11}(L_1NL_1 + L_1NL_1) = 2N_{11}(\frac{1}{2} U_{,x}W^2_{,x} + \frac{1}{8} U_{,x}V^2_{,x} - \frac{1}{4} U_{,x}V_{,x}U_{,y}$$

$$+ \frac{1}{8} U_{,x}U^2_{,y})$$

$$+ 2N_{11}W_{o,x}(\frac{1}{2} W^3_{,x} + \frac{1}{8} V^2_{,x}W_{,x} - \frac{1}{4} V_{,x}U_{,y}W_{,x}$$

$$+ \frac{1}{8} U^2_{,y}W_{,x}) \; ,$$

$$(N_{12} + N_{21})(L_1NL_2 + L_2NL_1) =$$

$$2N_{12}\Big(\frac{1}{2} U_{,x}W^2_{,y} - \frac{1}{R} U_{,x}VW_{,y} + \frac{1}{2R^2} U_{,x}V^2 - \frac{1}{8} U_{,x}V^2_{,x}$$
$$+ \frac{1}{4} U_{,x}V_{,x}U_{,y} - \frac{1}{8} U_{,x}U^2_{,y} + \frac{1}{2} V_{,y}W^2_{,y} + \frac{1}{8} V_{,y}V^2_{,x}$$
$$- \frac{1}{4} V_{,y}U_{,y} + \frac{1}{8} V_{,y}U^2_{,y} + \frac{1}{2R} WW^2_{,x} + \frac{1}{8R} WV^2_{,x}$$
$$- \frac{1}{4R} WV_{,x}U_{,y} + \frac{1}{8R} WU^2_{,y}\Big)$$
$$+ 2N_{12}W_{o,x}\Big(\frac{1}{2} W_{,x}W^2_{,y} - \frac{1}{R} V W_{,x}W_{,y} + \frac{1}{2R} V^2W_{,x}$$
$$- \frac{1}{8}V^2_{,x}W_{,x} + \frac{1}{4} U_{,y}V_{,x}W_{,x} - \frac{1}{8} U^2_{,y}W_{,x}\Big)$$
$$+ 2N_{12}W_{o,y}\Big(- \frac{1}{2R} VW^2_{,x} - \frac{1}{8R} VV^2_{,x} + \frac{1}{4R} U_{,y}VV_{,x}$$
$$- \frac{1}{8R} U^2_{,y}V + \frac{1}{2} W_{,y}W^2_{,x} + \frac{1}{8} V^2_{,x}W_{,y} - \frac{1}{4} U_{,y}V_{,x}W_{,y}$$
$$+ \frac{1}{8} U^2_{,y}W_{,y}\Big) ,$$

$$(N_{13} + N_{31})(L_1NL_3 + L_3NL_1) =$$

$$2N_{13}\Big(U_{,x}W_{,x}W_{,y} - \frac{1}{R} U_{,x}VW_{,x} + \frac{1}{2} V_{,x}W^2_{,x} + \frac{1}{8} V^3_{,x}$$
$$- \frac{1}{4} V^2_{,x}U_{,y} + \frac{1}{8} V_{,x}U^2_{,y} + \frac{1}{2} U_{,y}W^2_{,x} + \frac{1}{8} U_{,y}V^2_{,x}$$
$$- \frac{1}{4} V_{,x}U^2_{,y} + \frac{1}{8} U^3_{,y}\Big)$$
$$+ 2N_{13}W_{o,x}\Big(W^2_{,x}W_{,y} - \frac{1}{R} VW^2_{,x} + \frac{1}{2} W^2_{,x}W_{,y} + \frac{1}{8} V^2_{,x}W_{,y}$$
$$- \frac{1}{4} U_{,y}V_{,x}W_{,y} + \frac{1}{8} U^2_{,y}W_{,y} - \frac{1}{2R} VW^2_{,x} - \frac{1}{8R} VV^2_{,x}$$
$$+ \frac{1}{4R} U_{,y}VV_{,x} - \frac{1}{8R} U^2_{,y}V\Big)$$
$$+ 2N_{13}W_{o,y}\Big(\frac{1}{2} W^3_{,x} + \frac{1}{8} V^2_{,x}W_{,x} - \frac{1}{4} U_{,y}V_{,x}W_{,x} + \frac{1}{8} U^2_{,y}W_{,x}\Big) ,$$

$$(N_{14} + N_{41})(L_1NL_4 + L_4NL_1) =$$

$$2N_{14}(-\frac{1}{2}W_{,xx}W^2_{,x} - \frac{1}{8}W_{,xx}V^2_{,x} + \frac{1}{4}W_{,xx}V_{,x}U_{,y} - \frac{1}{8}W_{,xx}U^2_{,y}) \quad ,$$

$$(N_{15} + N_{51})(L_1NL_5 + L_5NL_1) =$$

$$2N_{15}(-\frac{1}{2}W_{,yy}W^2_{,x} - \frac{1}{8}W_{,yy}V^2_{,x} + \frac{1}{4}W_{,yy}V_{,x}U_{,y} - \frac{1}{8}W_{,yy}U^2_{,y}$$
$$+ \frac{1}{2R}V_{,y}W^2_{,x} + \frac{1}{8R}V_{,y}V^2_{,x} - \frac{1}{4R}V_{,y}V_{,x}U_{,y} + \frac{1}{8R}V_{,y}U^2_{,y})$$

$$(N_{16} + N_{61})(L_1NL_6 + L_6NL_1) =$$

$$2N_{16}(-W_{,xy}W^2_{,x} - \frac{1}{4}W_{,xy}V^2_{,x} + \frac{1}{2}W_{,xy}V_{,x}U_{,y} - \frac{1}{4}W_{,xy}U^2_{,y}$$
$$+ \frac{3}{4R}V_{,x}W^2_{,x} + \frac{3}{16R}V^3_{,x} - \frac{3}{8R}V^2_{,x}U_{,y} + \frac{3}{16R}V_{,x}U^2_{,y}$$
$$- \frac{1}{4R}U_{,y}W^2_{,x} - \frac{1}{16R}U_{,y}V^2_{,x} + \frac{1}{8R}V_{,x}U^2_{,y} - \frac{1}{16R}U^3_{,y}) \quad ;$$

$i = 2 \quad , \quad j = 2, 3, \ldots, 6$:

$$N_{22}(L_2NL_2 + L_2NL_2) = 2N_{22}(\frac{1}{2}V_{,y}W^2_{,y} - \frac{1}{R}V_{,y}VW_{,y} + \frac{1}{2R^2}V_{,y}V^2$$
$$- \frac{1}{8}V_{,y}V^2_{,x} + \frac{1}{4}V_{,y}V_{,x}U_{,y} - \frac{1}{8}V_{,y}U^2_{,y} + \frac{1}{2R}WW^2_{,y}$$
$$- \frac{1}{R^2}WVW_{,y} + \frac{1}{2R^3}WV^2 - \frac{1}{8}WV^2_{,x} + \frac{1}{4R}WV_{,x}U_{,y}$$
$$- \frac{1}{8R}WU^2_{,y}) \quad ,$$
$$+ 2N_{22}W_{o,y}(-\frac{1}{2R}VW^2_{,y} + \frac{1}{R^2}V^2W_{,y} - \frac{1}{2R^2}V^3 + \frac{1}{8R}VV^2_{,x}$$
$$- \frac{1}{4R}U_{,y}VV_{,x} + \frac{1}{8R}U^2_{,y}V + \frac{1}{2}W^3_{,y} - \frac{1}{R}VW^2_{,y}$$
$$+ \frac{1}{2R}V^2W_{,y} - \frac{1}{8}V^2_{,x}W_{,y} + \frac{1}{4}U_{,y}V_{,x}W_{,y} - \frac{1}{8}U^2_{,y}W_{,y}) \quad ,$$

$$(N_{23} + N_{32})(L_2NL_3 + L_3NL_2) =$$

$$\begin{aligned}
&2N_{23}\Big(V_{,y}W_{,x}W_{,y} - \frac{1}{R} V_{,y}VW_{,x} + \frac{1}{R} WW_{,x}W_{,y} - \frac{1}{R^2} WVW_{,x} \\
&\quad + \frac{1}{2} V_{,x}W^2_{,y} - \frac{1}{R} V_{,x}VW_{,y} + \frac{1}{2R^2} V_{,x}V^2 - \frac{1}{8} V^3_{,x} \\
&\quad + \frac{1}{4} V^2_{,x}U_{,y} - \frac{1}{8} V_{,x}U^2_{,y} + \frac{1}{2} U_{,y}W^2_{,y} - \frac{1}{R} U_{,y}VW_{,y} \\
&\quad + \frac{1}{2R^2} U_{,y}V^2 - \frac{1}{8} U_{,y}V^2_{,x} + \frac{1}{4} V_{,x}U^2_{,y} - \frac{1}{8} U^3_{,y}\Big) \quad , \\
&+ 2N_{23}W_{o,y}\Big(- \frac{1}{R} VW_{,x}W_{,y} + \frac{1}{R^2} V^2W_{,x} + W_{,x}W^2_{,y} - \frac{1}{R} VW_{,x}W_{,y} \\
&\quad + \frac{1}{2} W_{,x}W^2_{,y} - \frac{1}{R} VW_{,x}W_{,y} + \frac{1}{2R} V^2W_{,x} - \frac{1}{8} V^2_{,x}W_{,x} \\
&\quad + \frac{1}{4} U_{,y}V_{,x}W_{,x} - \frac{1}{8} U^2_{,y}W_{,x}\Big) \\
&+ 2N_{23}W_{o,x}\Big(\frac{1}{2} W^3_{,y} - \frac{1}{R} VW^2_{,y} + \frac{1}{2R} V^2W_{,y} - \frac{1}{8} V^2_{,x}W_{,y} \\
&\quad + \frac{1}{4} U_{,y}V_{,x}W_{,y} - \frac{1}{8} U^2_{,y}W_{,y} - \frac{1}{2R} VW^2_{,y} + \frac{1}{R^2} V^2W_{,y} \\
&\quad - \frac{1}{2R^2} V^3 + \frac{1}{8R} VV^2_{,x} - \frac{1}{4R} U_{,y}VV_{,x} + \frac{1}{8R} U^2_{,y}V\Big) \quad ,
\end{aligned}$$

$$(N_{24} + N_{42})(L_2NL_4 + N_4NL_2) =$$

$$\begin{aligned}
&2N_{24}\Big(- \frac{1}{2} W_{,xx}W^2_{,y} + \frac{1}{R} W_{,xx}VW_{,y} - \frac{1}{2R^2} W_{,xx}V^2 \\
&\quad + \frac{1}{8} W_{,xx}V^2_{,x} - \frac{1}{4} W_{,xx}V_{,x}U_{,y} + \frac{1}{8} W_{,xx}U^2_{,y}\Big) \quad ,
\end{aligned}$$

$$(N_{25} + N_{52})(L_2NL_5 + L_5NL_2) =$$

$$\begin{aligned}
&2N_{25}\Big(- \frac{1}{2} W_{,yy}W^2_{,y} + \frac{1}{R} W_{,yy}VW_{,y} - \frac{1}{2R^2} W_{,yy}V^2 \\
&\quad + \frac{1}{8} W_{,yy}V^2_{,x} - \frac{1}{4} W_{,yy}V_{,x}U_{,y} + \frac{1}{8} W_{,yy}U^2_{,y} + \frac{1}{2R} V_{,y}W^2_{,y} \\
&\quad - \frac{1}{R^2} VV_{,y}W_{,y} + \frac{1}{2R^3} V_{,y}V^2 - \frac{1}{8R} V_{,y}V^2_{,x} + \frac{1}{4R} V_{,y}V_{,x}U_{,y} \\
&\quad - \frac{1}{8R} V_{,y}U^2_{,y}\Big) \quad ,
\end{aligned}$$

$(N_{26} + N_{62})(L_2NL_6 + L_6NL_2) =$

$$2N_{26}(-W_{,xy}W^2_{,y} + \frac{2}{R} W_{,xy}VW_{,y} - \frac{1}{R^2} W_{,xy}V^2 + \frac{1}{4} W_{,xy}V^2_{,x}$$

$$- \frac{1}{2} W_{,xy}V_{,x}U_{,y} + \frac{1}{4} W_{,xy}U^2_{,y} + \frac{3}{4R} V_{,x}W^2_{,y}$$

$$- \frac{3}{2R^2} V_{,x}VW_{,y} + \frac{3}{4R^3} V_{,x}V^2 - \frac{3}{16R} V^3_{,x} + \frac{3}{8R} V^2_{,x}U_{,y}$$

$$- \frac{3}{16R} V_{,x}U_{,y}^{\;2} - \frac{1}{4R} U_{,y}W^2_{,y} + \frac{1}{2R^2} U_{,y}VW_{,y} - \frac{1}{4R^3} U_{,y}V^2$$

$$+ \frac{1}{16R} U_{,y}V^2_{,x} - \frac{1}{8R} V_{,x}U^2_{,y} + \frac{1}{16R} U^3_{,y}) \quad ;$$

$i = 3 \quad , \quad j = 3, 4, 5, 6:$

$N_{33}(L_3NL_3 + L_3NL_3) =$

$$2N_{33}(V_{,x}W_{,x}W_{,y} - VV_{,x}W_{,x}/R + U_{,y}W_{,x}W_{,y} - VU_{,y}W_{,x}/R)$$

$$+ 2N_{33}W_{o,x}(W_{,x}W^2_{,y} - \frac{2}{R} VW_{,x}W_{,y} + \frac{1}{R^2} V^2W_{,x})$$

$$+ 2N_{33}W_{o,y}(W^2_{,x}W_{,y} - \frac{1}{R} VW^2_{,x}) \quad ,$$

$(N_{34} + N_{43})(L_3NL_4 + L_4NL_3) =$

$$2N_{34}(- W_{,xx}W_{,x}W_{,y} + W_{,xx}VW_{,x}/R) \quad ,$$

$(N_{35} + N_{53})(L_3NL_5 + L_5NL_3) =$

$$2N_{35}(- W_{,yy}W_{,x}W_{,y} + W_{,yy}VW_{,x}/R + V_{,y}W_{,x}W_{,y}/R - V_{,y}VW_{,x}/R) \quad ,$$

$$(N_{36} + N_{63})(L_3NL_6 + L_6NL_3) =$$

$$2N_{36}(-2W_{,xy}W_{,x}W_{,y} + \frac{2}{R} W_{,xy}VW_{,x} + \frac{3}{2R} V_{,x}W_{,x}W_{,y}$$

$$- \frac{3}{2R^2} V_{,x}VW_{,x} - \frac{1}{2R} U_{,y}W_{,x}W_{,y} + \frac{1}{2R^2} U_{,y}VW_{,x})$$

All terms for which $i \geq 4$ and $j \geq 4$ are identically zero because NL_i are zero for these values of the index i.

The following unique, nonzero coefficients a_{ijk} can be identified. For convenience, they are grouped so as to be representative of the character of the product of displacement and/or derivatives with which they are associated.

WITHOUT IMPERFECTIONS

Group I (All factors in a product are the same).

3,3,3	$U_{,y}U_{,y}U_{,y}$:	$(N_{13} - N_{23})/4 + (-N_{16} + N_{26})/8R$
5,5,5	$V_{,x}V_{,x}V_{,x}$:	$(N_{13} - N_{23})/4 + 3(N_{16} - N_{26})/8R$

Group II (Two factors in a product are the same).

2,3,3	$U_{,x}U_{,y}U_{,y}$:	$(N_{11} - N_{12})/12$
2,4,4	$U_{,x}V\ V$:	$N_{12}/3R^2$
2,5,5	$U_{,x}V_{,x}V_{,x}$:	$(N_{11} - N_{12})/4$
2,8,8	$U_{,x}W_{,x}W_{,x}$:	$N_{11}/3$
2,9,9	$U_{,x}W_{,y}W_{,y}$:	$N_{12}/3$
3,3,5	$U_{,y}U_{,y}V_{,x}$:	$(-N_{13} + N_{23})/12 + 5(N_{16} - N_{26})/24R$
3,3,6	$U_{,y}U_{,y}V_{,y}$:	$(N_{12} - N_{22})/12 + (N_{15} - N_{25})/12R$
3,3,7	$U_{,y}U_{,y}W$:	$(N_{12} - N_{22})/12R$

3,3,10	$U_{,y}U_{,y}W_{,xx}$:	$(-N_{14} + N_{24})/12$
3,3,11	$U_{,y}U_{,y}W_{,xy}$:	$(-N_{16} + N_{26})/6$
3,3,12	$U_{,y}U_{,y}W_{,yy}$:	$(-N_{15} + N_{25})/12$
3,4,4	$U_{,y}V\ V$:	$N_{23}/3R^2 - N_{26}/6R^3$
3,5,5	$U_{,y}V_{,x}V_{,x}$:	$(-N_{13} + N_{23})/12 + 7(-N_{16} + N_{26})/24R$
3,8,8	$U_{,y}W_{,x}W_{,x}$:	$N_{13}/3 - N_{16}/6R$
3,9,9	$U_{,y}W_{,y}W_{,y}$:	$N_{23}/3 - N_{26}/6R$
4,4,5	$V\ V\ V_{,x}$:	$N_{23}/3R^2 + N_{26}/2R^3$
4,4,6	$V\ V\ V_{,y}$:	$N_{22}/3R^2 + N_{25}/3R^3$
4,4,7	$V\ V\ W$:	$N_{22}/3R^3$
4,4,10	$V\ V\ W_{,xx}$:	$-N_{24}/3R^2$
4,4,11	$V\ V\ W_{,xy}$:	$-2N_{26}/3R^2$
4,4,12	$V\ V\ W_{,yy}$:	$-N_{25}/3R^2$
5,5,6	$V_{,x}V_{,x}V_{,y}$:	$(N_{12} - N_{22})/12 + (N_{15} - N_{25})/12R$
5,5,7	$V_{,x}V_{,x}W$:	$(N_{12} - N_{22})/12R$
5,5,10	$V_{,x}V_{,x}W_{,xx}$:	$(-N_{14} + N_{24})/12$
5,5,11	$V_{,x}V_{,x}W_{,xy}$:	$(-N_{16} + N_{26})/6$
5,5,12	$V_{,x}V_{,x}W_{,yy}$:	$(-N_{15} + N_{25})/12$
5,8,8	$V_{,x}W_{,x}W_{,x}$:	$N_{13}/3 + N_{16}/2R$
5,9,9	$V_{,x}W_{,y}W_{,y}$:	$N_{23}/3 + N_{26}/2R$
6,8,8	$V_{,y}W_{,x}W_{,x}$:	$N_{12}/3 + N_{15}/3R$
6,9,9	$V_{,y}W_{,y}W_{,y}$:	$N_{22}/3 + N_{25}/3R$
7,8,8	$W\ W_{,x}W_{,x}$:	$N_{12}/3R$
7,9,9	$W\ W_{,y}W_{,y}$:	$N_{22}/3R$
8,8,10	$W_{,x}W_{,x}W_{,xx}$:	$-N_{14}/3$

8,8,11	$W_{,x}W_{,x}W_{,xy}$:	$-2N_{16}/3$
8,8,12	$W_{,x}W_{,x}W_{,yy}$:	$-N_{15}/3$
9,9,10	$W_{,y}W_{,y}W_{,xx}$:	$-N_{24}/3$
9,9,11	$W_{,y}W_{,y}W_{,xy}$:	$-2N_{26}/3$
9,9,12	$W_{,y}W_{,y}W_{,yy}$:	$-N_{25}/3$

<u>Group III</u> (All factors in the product are unique).

2,3,5	$U_{,x}U_{,y}V_{,x}$:	$(-N_{11} + N_{12})/12$
2,4,8	$U_{,x}V\ W_{,x}$:	$-N_{13}/3R$
2,4,9	$U_{,x}V\ W_{,y}$:	$-N_{12}/3R$
2,8,9	$U_{,x}W_{,x}W_{,y}$:	$N_{13}/3$
3,4,8	$U_{,y}V\ W_{,x}$:	$-N_{33}/3R + N_{36}/6R^2$
3,5,9	$U_{,y}V\ W_{,y}$:	$-N_{23}/3R + N_{26}/6R^2$
3,5,6	$U_{,y}V_{,x}V_{,y}$:	$(-N_{12} + N_{22})/12 + (-N_{15} + N_{25})/12R$
3,5,7	$U_{,y}V_{,x}W$:	$(-N_{12} + N_{22})/12R$
3,5,10	$U_{,y}V_{,x}W_{,xx}$:	$(N_{14} - N_{24})/12$
3,5,11	$U_{,y}V_{,x}W_{,xy}$:	$(N_{16} - N_{26})/6$
3,5,12	$U_{,y}V_{,x}W_{,yy}$:	$(N_{15} - N_{25})/12$
3,8,9	$U_{,y}W_{,x}W_{,y}$:	$N_{33}/3 - N_{36}/6R$
4,5,8	$V\ V_{,x}W_{,x}$:	$-N_{33}/3R - N_{36}/2R^2$
4,5,9	$V\ V_{,x}W_{,y}$:	$-N_{23}/3R - N_{26}/2R^2$
4,6,8	$V\ V_{,y}W_{,x}$:	$-N_{23}/3R - N_{35}/3R^2$
4,6,9	$V\ V_{,y}W_{,y}$:	$-N_{22}/3R - N_{25}/3R^2$
4,7,8	$V\ W\ W_{,x}$:	$-N_{23}/3R^2$
4,7,9	$V\ W\ W_{,y}$:	$-N_{22}/3R^2$
4,8,10	$V\ W_{,x}W_{,xx}$:	$N_{34}/3R$

4,8,11	$V\ W_{,x}W_{,xy}$:	$2N_{36}/3R$
4,8,12	$V\ W_{,x}W_{,yy}$:	$N_{35}/3R$
4,9,10	$V\ W_{,y}W_{,xx}$:	$N_{24}/3R$
4,9,11	$V\ W_{,y}W_{,xy}$:	$2N_{26}/3R$
4,9,12	$V\ W_{,y}W_{,yy}$:	$N_{25}/3R$
5,8,9	$V_{,x}W_{,x}W_{,y}$:	$N_{33}/3 + N_{36}/2R$
6,8,9	$V_{,y}W_{,x}W_{,y}$:	$N_{23}/3 + N_{35}/3R$
7,8,9	$W\ W_{,x}W_{,y}$:	$N_{23}/3R$
8,9,10	$W_{,x}W_{,y}W_{,xx}$:	$-N_{34}/3$
8,9,11	$W_{,y}W_{,y}W_{,xy}$:	$-2N_{36}/3$
8,9,12	$W_{,x}W_{,y}W_{,yy}$:	$-N_{35}/3$

The coefficients have been symmetrized by dividing Groups I, II, and III by 1, 3, and 6, respectively.

CONTRIBUTIONS DUE TO IMPERFECTIONS

Group I (All factors in a produce are the same.)

8,8,8	$W_{,x}W_{,x}W_{,x}$:	$N_{11}W_{o,x} + N_{13}W_{o,y}$
4,4,4	$V\ V\ V$:	$-\frac{N_{22}}{R^3}W_{o,y} - \frac{N_{23}}{R^3}W_{o,x}$
9,9,9	$W_{,y}W_{,y}W_{,y}$:	$N_{22}W_{o,y} + N_{23}W_{o,x}$

Group II (Two factors in a product are the same.)

5,5,8	$V_{,x}V_{,x}W_{,x}$:	$\frac{1}{12}\{(N_{11} - N_{12})W_{o,x} + (N_{13} - N_{23})W_{o,y}\}$
3,3,8	$U_{,y}U_{,y}W_{,x}$:	$\frac{1}{12}\{(N_{11} - N_{12})W_{o,x} + (N_{13} - N_{23})W_{o,y}\}$

8,9,9	$W_{,x}W_{,y}W_{,y}$:	$\frac{1}{3}N_{12}W_{o,x} + N_{23}W_{o,y} + \frac{2}{3}N_{33}W_{o,x}$
4,4,8	$V\ V\ W_{,x}$:	$\frac{N_{12}}{3R^2}W_{o,x} + \frac{N_{23}}{R^2}W_{o,y} + \frac{2N_{33}}{3R^2}W_{o,x}$
4,8,8	$V\ W_{,x}W_{,x}$:	$-\frac{N_{12}}{3R}W_{o,y} - \frac{N_{13}}{R}W_{o,x} - \frac{2N_{33}}{3R}W_{o,y}$
4,5,5	$V\ V_{,x}V_{,x}$:	$\frac{1}{12R}\{(N_{22} - N_{12})W_{o,y} + (N_{23} - N_{13})W_{o,x}\}$
3,3,4	$U_{,y}U_{,y}V$:	$\frac{1}{12R}\{(N_{22} - N_{12})W_{o,y} + (N_{23} - N_{13})W_{o,x}\}$
8,8,9	$W_{,x}W_{,x}W_{,y}$:	$\frac{N_{12}}{3}W_{o,y} + N_{13}W_{o,x} + \frac{2}{3}N_{33}W_{o,y}$
5,5,9	$V_{,x}V_{,x}W_{,y}$:	$\frac{1}{12}\{-(N_{22} - N_{12})W_{o,y} + (N_{13} - N_{23})W_{o,x}\}$
3,3,9	$U_{,y}U_{,y}W_{,y}$:	$\frac{1}{12}\{-(N_{22} - N_{12})W_{o,y} + (N_{13} - N_{23})W_{o,x}\}$
4,9,9	$V\ W_{,y}W_{,y}$:	$-\frac{N_{22}}{R}W_{o,y} - \frac{N_{23}}{R}W_{o,x}$
4,4,9	$V,\ V,\ W_{,y}$:	$\frac{N_{22}}{R^2}W_{o,y} + \frac{N_{23}}{R^2}W_{o,x}$

<u>Group III</u> (All factors in a product are different.)

3,5,8 $\quad U_{,y}V_{,x}W_{,x}$: $\frac{1}{12}\{-(N_{11} - N_{12})W_{o,x} - (N_{13} - N_{23})W_{o,y}\}$

4,8,9 $\quad V\ W_{,x}W_{,y}$: $-\frac{N_{12}}{3R} W_{o,x} - \frac{N_{23}}{R} W_{o,y} - \frac{2N_{33}}{3R} W_{o,x}$

3,4,5 $\quad U_{,y}V\ V_{,x}$: $\frac{1}{12R}\{-(N_{22} - N_{12})W_{o,y} + (N_{13} - N_{23})W_{o,x}\}$

3,5,9 $\quad U_{,y}V_{,x}W_{,y}$: $\frac{1}{12}\{(N_{22} - N_{12})W_{o,y} - (N_{13} - N_{23})W_{o,x}\}$

The foregoing coefficients have been symmetrized by dividing Groups I, II, and III by 1, 3, and 6, respectively.

<u>Quartic Contributions</u>. The following terms result from an expansion of the quartic portion of the strain energy of an element.

$i = 1 \quad , \quad j = 1, 2, \ldots, 6:$

$$
\begin{aligned}
N_{11}NL_1NL_1 = N_{11}\Big(&\frac{1}{4} W^4_{,x} + \frac{1}{64} V^4_{,x} + \frac{1}{16} V^2_{,x}U^2_{,y} + \frac{1}{64} U^4_{,y} \\
&+ \frac{1}{8} W^2_{,x}V^2_{,x} - \frac{1}{16} V^3_{,x}U_{,y} - \frac{1}{16} V_{,x}U^3_{,y} \\
&+ \frac{1}{8} W^2_{,x}U^2_{,y} - \frac{1}{4} W^2_{,x}V_{,x}U_{,y} + \frac{1}{32} V^2_{,x}U^2_{,y}\Big) \quad ,
\end{aligned}
$$

$(N_{12} + N_{21})NL_1NL_2 =$

$$
\begin{aligned}
2N_{12}\Big(&\frac{1}{4} W^2_{,x}W^2_{,y} + \frac{1}{16} W^2_{,y}V^2_{,x} - \frac{1}{8} W^2_{,y}V_{,x}U_{,y} \\
&+ \frac{1}{16} W^2_{,y}U^2_{,y} - \frac{1}{2R} W^2_{,x}VW_{,y} - \frac{1}{8R} V^2_{,x}VW_{,y} + \frac{1}{4R} VW_{,y}V_{,x}U_{,y} \\
&- \frac{1}{8R} U^2_{,y}VW_{,y} + \frac{1}{4R^2} V^2W^2_{,x} + \frac{1}{16R^2} V^2V^2_{,x} \\
&- \frac{1}{8R^2} V^2V_{,x}U_{,y} + \frac{1}{16R^2} V^2U^2_{,y} + \frac{1}{8} W^2_{,x}V_{,x}U_{,y} \\
&+ \frac{1}{32} V^3_{,x}U_{,y} - \frac{1}{16} V^2_{,x}U^2_{,y} + \frac{1}{32} V_{,x}U^3_{,y} - \frac{1}{16} W^2_{,x}U^2_{,y} \\
&- \frac{1}{64} V^2_{,x}U^2_{,y} + \frac{1}{32} V_{,x}U^3_{,y} - \frac{1}{64} U^4_{,y} - \frac{1}{16} V^2_{,x}W^2_{,x} \\
&- \frac{1}{64} V^4_{,x} + \frac{1}{32} V^3_{,x}U_{,y} - \frac{1}{64} V^2_{,x}U^2_{,y}\Big) \quad ,
\end{aligned}
$$

$(N_{13} + N_{31})NL_1NL_3 =$

$$
\begin{aligned}
2N_{13}\Big(&\frac{1}{2} W^3_{,x}W_{,y} + \frac{1}{8} V^2_{,x}W_{,x}W_{,y} - \frac{1}{4} V_{,x}U_{,y}W_{,x}W_{,y} \\
&+ \frac{1}{8} U^2_{,y}W_{,x}W_{,y} - \frac{1}{2R} W^3_{,x}V - \frac{1}{8R} V^2_{,x} VW_{,x} \\
&+ \frac{1}{4R} V_{,x}U_{,y}VW_{,x} - \frac{1}{8R} U^2_{,y}VW_{,x}\Big) \quad ,
\end{aligned}
$$

$(N_{14} + N_{41})NL_1NL_4 = 0 \quad ,$

$(N_{15} + N_{51})NL_1NL_5 = 0 \quad ,$

$(N_{16} + N_{61})NL_1NL_6 = 0 \quad ;$

$i = 2 \quad , \quad j = 2, 3, \ldots, 6:$

$$N_{22}\ NL_2NL_2 = N_{22}(\frac{1}{4} W^4_{,y} + \frac{1}{R^2} V^2W^2_{,y} + \frac{1}{4R^4} V^4 + \frac{1}{64} V^4_{,x}$$
$$+ \frac{1}{16} V^2_{,x}U^2_{,y} + \frac{1}{64} U^4_{,y} - \frac{1}{R} VW^3_{,y} + \frac{1}{2R^2} V^2W^2_{,y}$$
$$- \frac{1}{8} V^2_{,x}W^2_{,y} + \frac{1}{4} W^2_{,y}V_{,x}U_{,y} - \frac{1}{8} W^2_{,y}U^2_{,y}$$
$$- \frac{1}{R^3} V^3W_{,y} + \frac{1}{4R} VW_{,y}V^2_{,x} - \frac{1}{2R} VW_{,y}V_{,x}U_{,y}$$
$$+ \frac{1}{4R} VW_{,y}U^2_{,y} - \frac{1}{8R^2} V^2V^2_{,x} + \frac{1}{4R^2} V^2V_{,x}U_{,y}$$
$$- \frac{1}{8R^2} V^2U^2_{,y} - \frac{1}{16} V^3_{,x}U_{,y} + \frac{1}{32} V^2_{,x}U^2_{,y}$$
$$- \frac{1}{16} V_{,x}U^3_{,y}) \quad ,$$

$$(N_{23} + N_{32})NL_2NL_3 =$$
$$2N_{23}(\frac{1}{2} W_{,x}W^3_{,y} - \frac{1}{R} VW_{,x}W^2_{,y} + \frac{1}{2R^2} V^2W_{,x}W_{,y}$$
$$- \frac{1}{8} V^2_{,x}W_{,x}W_{,y} + \frac{1}{4} V_{,x}U_{,y}W_{,x}W_{,y} - \frac{1}{8} U^2_{,y}W_{,x}W_{,y}$$
$$- \frac{1}{2R} W^2_{,y}VW_{,x} + \frac{1}{R^2} V^2W_{,x}W_{,y} - \frac{1}{2R^3} V^3W_{,x}$$
$$+ \frac{1}{8R} V^2_{,x}VW_{,x} - \frac{1}{4R} V_{,x}U_{,y}VW_{,x} + \frac{1}{8R} U^2_{,y}VW_{,x}) \quad ,$$

$$(N_{24} + N_{42})NL_2NL_4 = 0 \quad ,$$

$$(N_{25} + N_{52})NL_2NL_5 = 0 \quad ,$$

$$(N_{26} + N_{62})NL_2NL_6 = 0 \quad ;$$

$i = 3 \quad , \quad j = 3, 4, 5, 6:$

$$N_{33}NL_3NL_3 = N_{33}(W^2_{,x}W^2_{,y} - \frac{2}{R} W^2_{,x}VW_{,y} + \frac{1}{R^2} V^2W^2_{,x}) \quad ,$$

$$(N_{34} + N_{43})NL_3NL_4 = 0 \quad ,$$

$$(N_{35} + N_{53})NL_3NL_5 = 0 \quad ,$$

$$(N_{36} + N_{63})NL_3NL_6 = 0 \quad .$$

All terms for which $i \geq 4$ and $j \geq 4$ are identically zero because NL_i are zero for these values of the index i.

The following unqiue, nonzero coefficients a_{ijkl} can be identified. For convenience they are grouped so as to be representative of the character of the product of displacement and/or derivatives with which they are associated.

Group I (All factors in a product are the same.)

8,8,8,8	$W_{,x}W_{,x}W_{,x}W_{,x}$:	$N_{11}/4$
5,5,5,5	$V_{,x}V_{,x}V_{,x}V_{,x}$:	$(N_{11} - 2N_{12} + N_{22})/64$
9,9,9,9	$W_{,y}W_{,y}W_{,y}W_{,y}$:	$N_{22}/4$
4,4,4,4	$V\ V\ V\ V$:	$N_{22}/4R^4$
3,3,3,3	$U_{,y}U_{,y}U_{,y}U_{,y}$:	$(N_{11} - 2N_{12} + N_{22})/64$

Group II (Three factors in a product are the same.)

4,9,9,9	$V W_{,y} W_{,y} W_{,y}$:	$-N_{22}/4R$
4,4,4,9	$V\ V\ V\ W_{,y}$:	$-N_{22}/4R^3$
3,5,5,5	$U_{,y} V_{,x} V_{,x} V_{,x}$:	$(-N_{11} + 2N_{12} - N_{22})/64$
4,4,4,8	$V\ V\ V\ W_{,x}$:	$-N_{23}/4R^3$
3,3,3,5	$U_{,y} U_{,y} U_{,y} V_{,x}$:	$(-N_{11} + 2N_{12} - N_{22})/64$
8,8,8,9	$W_{,x} W_{,x} W_{,x} W_{,y}$:	$N_{13}/4$
8,9,9,9	$W_{,x} W_{,y} W_{,y} W_{,y}$:	$N_{23}/4$
4,8,8,8	$V\ W_{,x} W_{,x} W_{,x}$:	$-N_{13}/4R$

Group III (Two factors in a product are the same: the other two are different.)

8,8,3,5	$W_{,x} W_{,x} U_{,y} V_{,x}$:	$(-N_{11} + N_{12})/48$
9,9,3,5	$W_{,y} W_{,y} U_{,y} V_{,x}$:	$(-N_{12} + N_{22})/48$
5,5,4,9	$V_{,x} V_{,x} V\ W_{,y}$:	$(-N_{12} + N_{22})/48R$
3,3,4,9	$U_{,y} U_{,y} V\ W_{,y}$:	$(-N_{12} + N_{22})/48R$
4,4,3,5	$V\ V\ U_{,y} V_{,x}$:	$(-N_{12} + N_{22})/48R^2$
8,8,4,9	$W_{,x} W_{,x} V\ W_{,y}$:	$-(N_{12} + 2N_{33})/12R$
5,5,8,9	$V_{,x} V_{,x} W_{,x} W_{,y}$:	$(N_{13} - N_{23})/48$
4,4,8,9	$V\ V\ W_{,x} W_{,y}$:	$N_{23}/4R^2$
9,9,4,8	$W_{,y} W_{,y} V\ W_{,x}$:	$-N_{23}/4R$
5,5,4,8	$V_{,x} V_{,x} V\ W_{,x}$:	$(-N_{13} + N_{23})/48R$
3,3,8,9	$U_{,y} U_{,y} W_{,x} W_{,y}$:	$(N_{13} - N_{23})/48$
3,3,4,9	$U_{,y} U_{,y} V\ W_{,x}$:	$(-N_{13} + N_{23})/48R$

Group IV (Two different pairs of factors in a product.)

5,5,8,8,	$V_{,x}V_{,x}W_{,x}W_{,x}$:	$(N_{11} - N_{12})/48$
3,3,8,8	$U_{,y}U_{,y}W_{,x}W_{,x}$:	$(N_{11} - N_{12})/48$
4,4,9,9	$V\ V\ W_{,y}W_{,y}$:	$N_{22}/4R^2$
5,5,3,3	$V_{,x}V_{,x}U_{,y}U_{,y}$:	$3(N_{11} - 2N_{12} + N_{22})/192$
5,5,9,9	$V_{,x}V_{,x}W_{,y}W_{,y}$:	$(N_{12} - N_{22})/48$
3,3,9,9	$U_{,y}U_{,y}W_{,y}W_{,y}$:	$(N_{12} - 2N_{22})/48$
4,4,5,5	$V\ V\ V_{,x}V_{,x}$:	$(N_{12} - N_{22})/48R^2$
4,4,3,3	$V\ V\ U_{,y}U_{,y}$:	$(N_{12} - N_{22})/48R^2$
8,8,9,9	$W_{,x}W_{,x}W_{,y}W_{,y}$:	$N_{12}/12 + N_{33}/6$
4,4,8,8	$V\ V\ W_{,x}W_{,x}$:	$(\frac{1}{2}N_{12} + N_{22})/6R^2$

Group V (All factors in a product are different.)

3,4,5,9	$U_{,y}V\ V_{,x}W_{,y}$:	$(N_{12} - N_{22})/48R$
3,5,8,9	$U_{,y}V_{,x}W_{,x}W_{,y}$:	$(-N_{13} + N_{23})/48$
3,4,5,8	$U_{,y}V\ V_{,x}W_{,x}$:	$(N_{13} - N_{23})/48R$

The coefficients a_{ijkl} are completely symmetrized by dividing Groups I, II, III, IV, and V by 1, 4, 6, 12, and 24, respectively.

APPENDIX B

FINITE-DIFFERENCE FORMULAS

Finite-difference formulas required to express the displacement components and their derivatives at the centroid of an element are described below. One-dimensional representation for function value and its first and second order derivatives are given for both the x- and y- directions. The procedure used to obtain two-dimensional centroidal function values and first and second order partial derivatives from the one-dimensional formulas is also described.

The symbol $(\tilde{\ })$ is used to denote a centroidal quantity and f_i signifies function values at nodal stations for one-dimensional expressions, while g_i signifies function values at nodal stations associated with the two-dimensional finite-difference grid. Finally, the notations $(\ )'$ and $(\ )$ signify differentiation with respect to x or y, respectively.

ONE-DIMENSIONAL FINITE-DIFFERENCE FORMULAS

<u>Forward Difference Formulas: X-direction.</u>

$$\tilde{f} = a_1 f_{i+2} + a_2 f_{i+1} + a_3 f_i, \qquad \text{(B-1a)}$$

$$\tilde{f}' = a_4 f_{i+2} + a_5 f_{i+1} + a_6 f_i, \qquad \text{(B-1b)}$$

$$\tilde{f}'' = a_7 f_{i+2} + a_8 f_{i+1} + a_9 f_i; \qquad \text{(B-1c)}$$

where

$$a_1 = -\frac{3}{16}\,\frac{h^2}{k(h+k)}\,, \qquad a_4 = -\frac{1}{2}\,\frac{h}{k(h+k)}\,, \qquad a_7 = \frac{2}{k(h+k)}\,,$$

$$a_2 = \frac{1}{4}\,\frac{k+3/4h}{k}\,, \qquad a_5 = \frac{k+\frac{1}{2}h}{hk}\,, \qquad a_8 = -\frac{2}{hk}\,,$$

$$a_3 = \frac{3}{4}\,\frac{k+3/4h}{h+k}\,, \qquad a_6 = -\frac{k+3/2h}{h(h+k)}\,, \qquad a_9 = \frac{2}{h(h+k)}\,. \qquad \text{(B-2)}$$

Formulas (B-1) are obtained via a Taylor series expansion about the centroid of the shaded area of Figure (B-1)

<u>Backward Difference Formulas: x-direction</u>

$$\tilde{f} = b_1 f_{i-2} + b_2 f_{i-1} + b_3 f_i, \qquad \text{(B-3a)}$$

$$\dot{\tilde{f}} = -\,b_4 f_{i-2} - b_5 f_{i-1} - b_6 f_i, \qquad \text{(B-3b)}$$

$$\ddot{\tilde{f}} = b_7 f_{i-2} + b_8 f_{i-1} + b_9 f_i\,. \qquad \text{(B-3c)}$$

The b_i are obtained from formulas (B-2) by interchanging h and k.

<u>Forward Difference Formulas: y-direction.</u>

$$\tilde{f} = c_1 f_{j-2} + c_2 f_{j-1} + c_3 f_j\,, \qquad \text{(B-4a)}$$

$$\tilde{f}' = c_4 f_{j-2} + c_5 f_{j-1} + c_6 f_j, \qquad \text{(B-4b)}$$

$$\tilde{f}'' = c_7 f_{j-2} + c_8 f_{j-1} + c_9 f_j \, . \qquad \text{(B-4c)}$$

The c_i are obtained from formulas (B-2) by setting $h = \ell$ and $k = m$.

<u>Backward Difference Formulas: y-direction.</u>

$$\tilde{f} = d_1 f_{j-2} + d_2 f_{j-1} + d_3 f_j \, , \qquad \text{(B-5a)}$$

$$\dot{\tilde{f}} = -d_4 f_{j-2} - d_5 f_{j-1} - d_6 f_j , \qquad \text{(B-5b)}$$

$$\ddot{\tilde{f}} = d_7 f_{j-2} + d_8 f_{j-1} + d_9 f_j \, . \qquad \text{(B-5c)}$$

The d_j are obtained from formulas (B-2) by setting $h = m$ and $k = \ell$.

Central Difference Formulas: x-direction.

$$\tilde{f} = e_1 f_{i+1} + e_2 f_i + e_3 f_{i-1}\ , \qquad \text{(B-6a)}$$

$$\tilde{f}' = e_4 f_{i+1} + e_5 f_i + e_6 f_{i-1}\ , \qquad \text{(B-6b)}$$

$$\tilde{f}'' = e_7 f_{i+1} + e_8 f_i + e_9 f_{i-1}; \qquad \text{(B-6c)}$$

where:

$$e_1 = \frac{(k-h)(3h+k)}{16k(h+k)}\ , \quad e_4 = \frac{1}{2k}\ , \quad e_7 = \frac{2}{k(h+k)}\ ,$$

$$e_2 = \frac{(h+3k)(3h+k)}{16\ hk}, \quad e_5 = \frac{1}{2h} - \frac{1}{2k}, \quad e_8 = -\frac{2}{hk}. \qquad \text{(B-7)}$$

$$e_3 = -\frac{(k-h)(3k+h)}{16h\ (h+k)}, \quad e_6 = -\frac{1}{2h}, \quad e_9 = \frac{2}{h(h+k)}.$$

Central Difference Formulas: y-direction.

$$\tilde{f} = f_1 f_{j+1} + f_2 f_j + f_3 f_{j-1}\ , \qquad \text{(B-8a)}$$

$$\dot{\tilde{f}} = f_4 f_{j+1} + f_5 f_j + f_6 f_{j-1}\ , \qquad \text{(B-8b)}$$

$$\ddot{\tilde{f}} = f_7 f_{j+1} + f_8 f_j + f_9 f_{j-1}\ . \qquad \text{(B-8c)}$$

The f_j are obtained from the coefficients given in Eqs. (B-7) by setting $h = \ell$ and $k = m$.

Two-Dimensional Finite Difference Formulas

Two-dimensional finite-difference formulas are not developed explicitly for all 9 types of elements. However, the procedure used to obtain them is demonstrated explicitly for the centroidal function value for an exterior boundary element. First and second order partial derivatives are obtained in a similar manner.

The two-dimensional function value for the left exterior boundary element shown in Figure 7 is developed as follows. The one-dimensional centroidal function value expressed in terms of discrete values at the intersections of a horizonal line through the centroid with the finite-difference grid lines in the y-direction is:

$$\tilde{f} = a_1\, f_{i+2} + a_2\, f_{i+1} + a_3 f_i. \qquad \text{(B-9)}$$

Applying Eq. (B-8a) along lines i+2, i+1, and i, successively, yields:

$$\begin{aligned}\tilde{f} = a_1\, &\{f_1\, g_{i+2,j+1} + f_2 g_{i+2,j} + f_3 g_{i+2,j-1}\} \\ &+ a_2\{f_1 g_{i+1,j+1} + f_2 g_{i+1,j} + f_3 g_{i+1,j-1}\} \qquad \text{(B-10)} \\ &+ a_3\{f_1 g_{i,j+1} + f_2 g_{i,j} + f_3 g_{i,j-1}\}.\end{aligned}$$

According to the adopted element numbering scheme (Figure 7) the quantities g_{ij} can be designated by g_i, $i = 1,2, ---, 9$. Thus,

$$\tilde{f} = a_1 f_1 g_1 + a_2 f_2 g_2 + a_3 f_2 g_3 + a_2 f_1 g_4$$
$$+ a_2 f_3 g_5 + a_3 f_3 g_6 + a_3 f_1 g_7 + a_1 f_1 g_8 + a_1 f_3 g_9, \qquad \text{(B-11)}$$

or

$$\tilde{f} = b_{1i}\, g_i \quad ; \quad i = 1,2, ---, 9. \qquad \text{(B-12a)}$$

The definitions of the b_{1j} are obvious from a comparison of Eqs. (B-11) and (B-12).

Now first and second order partial derivatives are obtained in the same manner, hence:

$$\tilde{f}' = b_{2i} g_i, \qquad \text{(B-12b)}$$
$$\dot{\tilde{f}} = b_{3i} g_i, \qquad \text{(B-12c)}$$
$$\tilde{f}'' = b_{4i} g_i, \qquad \text{(B-12d)}$$
$$\dot{\tilde{f}}' = b_{5i} g_i, \qquad \text{(B-12e)}$$

and

$$\ddot{\tilde{f}} = b_{6i} g_i. \qquad \text{(B-12f)}$$

According to Eqs. (B-12) one obtains a transformation of the form:

$$[\tilde{f}] = [b][g]. \qquad \text{(B-13)}$$

APPENDIX C
USER GUIDE

This appendix contains information describing the cards that must be prepared by the program user and the output information to be expected.

INPUT FORMATION

Information to be provided by the user, and the format in which it must appear, is presented in this section.

1. RUN CARD (I5)

   Columns 1-5 Number of the run. (IRUN)

   It may be convenient to generate the load-deflection curve in distinct segments because approximate computer time to make a complete analysis is unknown. IRUN is an index that signifies the number of the segment under consideration.

   IRUN = 1 first run
   2 second run - continuation of the first run.
   .
   .
   .

2. TITLE CARD (18A4)

   Columns 1-72 Problem Identification

3. CONTROL CARD (2I10,3I5)

   Columns 1-10 Number of elements (NEL = 244 maximum)

   11-20 Number of nodal points (NUMNP = 240 maximum)

   21-25 Index that indicates the absence or presence of initial imperfection.

$$IEX = \begin{cases} 0 \text{ without initial imperfections} \\ 1 \text{ with initial imperfections} \end{cases}$$

26-30 Index that selects the linear bifurcation analysis.

$$IBIF = \begin{cases} 0 & \text{nonlinear analysis} \\ 1 & \text{linear bifurcation analysis only a} \end{cases}$$

31-35 Index that indicates whether the structure is flat plate or curved panel.

$$NFLAT = \begin{cases} 0 & \text{flat plate} \\ 1 & \text{curved panel} \end{cases}$$

4. RADIUS/CONVERGENCE CARD (2F10.0)

Columns 1-10 Radius of curvature in inches (R)

11-20 Convergence criterion for the modified Newton-Raphson iterative procedure. Program terminates if convergence at any load level has not occurred after ITMAX iterations. EPSI specifies acceptable increment of displacements.

5. IMPERFECTIONS CARD (5F10.0)

Columns 1-10 Amplitude W_o, of the initial imperfection.

11-20 Wave number in x-direction.

21-30 Wave number in y-direction.

31-40 x-coordinate of the lower left-hand corner of panel.

41-50 y-coordinate of the lower left-hand corner of panel.

One card is required. A blank card is required if there are no initial imperfections.

6. NODAL NUMBERING SCHEME/CUTOUT CARD (8I5)

Columns 1-5	NSCHM	= 1 Nodes numbered consecutively in the y-direction ≠ 1 Nodes numbered consecutively in the x-direction
6-10	ICTOUT	= 1 Panel has a cutout ≠ 1 Panel does not have a cutout
11-15	NROWS	Number of rows of node points
16-20	NCOLS	Number of columns of node points
21-25	IROW1	Row number of the row in which the side of the cutout nearest the row-baseline for the panel lies.
26-30	IROW2	Row number of the row in which the side of the cutout furtherest from the row-baseline for the panel lies.
31-35	ICOL1	Column number of the column in which the side of the cutout nearest the column-baseline for the panel lies.

7. ELEMENT TYPE CARD (10I5)

Element type designators for 10 elements appear on each card.

ITYPE = 1 Left exterior or right interior boundary element.

ITYPE = 2 Top exterior or bottom interior boundary element.

ITYPE = 3 Right exterior or left interior boundary element.

ITYPE = 4 Bottom exterior or top interior boundary element.

ITYPE = 5 Interior boundary element.

ITYPE = 6 Upper left corner element.

ITYPE = 7 Upper right corner element.

ITYPE = 8 Lower left corner element.

ITYPE = 9 Lower right corner element.

8. ELEMENT-NODAL POINT COORDINATION CARDS (10I5)

Columns	1-5	Primary nodal point associated with area element M = 1.
	6-10	Primary nodal point associated with area element M = 2.
	.	
	.	
	.	
	46-50	Primary nodal point associated with area element M = 10.

A card is required for every 10 area-elements. Last card may contain data for less than 10 area-elements. If there is no cut-out then an area-element label number and its primary node point label number are identical.

9. NODAL COORDINATE CARD (I5,2F10.0)

Columns	1-5	Nodal point number (N)
	6-15	x-coordinate of nodal point N (inches).
	16-25	y-coordinate of nodal point N (inches).

One card for each nodal point is required.

10. ELEMENT LOAD CARD (I5,3F10.0)

Columns	1-5	Element number (M)
	6-15	Centroidal load intensity in the z-direction

(normal to reference surface), lb/in^2.

16-25 Centroidal load intensity in the x-direction (along a generator), lb/in^2.

26-35 Centroidal load intensity in the y-direction (along a cross section circle), lb/in^2.

One card is required for each area element. The program multiplies the load intensity at the centroid of the area element by the element area and assumes the resultant acts at the centroid of the area element.

11. LOAD INCREMENT CARD (3F10.0)

Columns	1-10	ZINCR - Load Increment in z-direction.
	11-20	XINCR - Load Increment in x-direction.
	21-30	YINCR - Load Increment in y-direction.

12. BOUNDARY CONDITON CARD (I5,3I5)

Columns	1-5	Element number (M).
	6-10	Boundary condition index for the z degree of freedom.
	11-15	Boundary condition index for the x degree of freedom.
	16-20	Boundary condition index for the y degree of freedom.

One card is required for each element (including interior elements). The boundary condition index has the following meanings:

$$\text{Boundary condition index} = \begin{cases} 0 & \text{zero force} \\ 1 & \text{zero displacement} \end{cases}$$

The zero force boundary condition implies that either Q_n, N_n, N_{nt} (generalized transverse shear force, normal force, or shearing force) is zero according as

AD-A071 649 DAYTON UNIV OHIO F/G 20/11

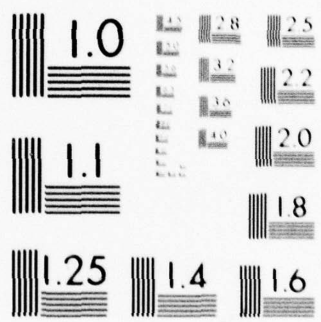
COLLAPSE LOAD ANALYSIS FOR PLATES AND SHELLS. (U)

MAY 79 N R BAULD, K SATYAMURTHY F33615-76-C-3145

UNCLASSIFIED AFFDL-TR-79-3038 NL

2 OF 3

AD A071649



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

to whether it applies to the z, x, or y degree of freedom.

13. NUMBER OF LAYERS CARD (I5)

Columns	1-5	Total number of layers (KN).

14. LAYER POSITION CARDS (F10.0)

Columns	1-10	Distance (inches) of the surface nearest the center of curvature from the reference surface for each layer. First layer is considered to be the one nearest the center of curvature. Distances measured toward the center of curvature are negative.

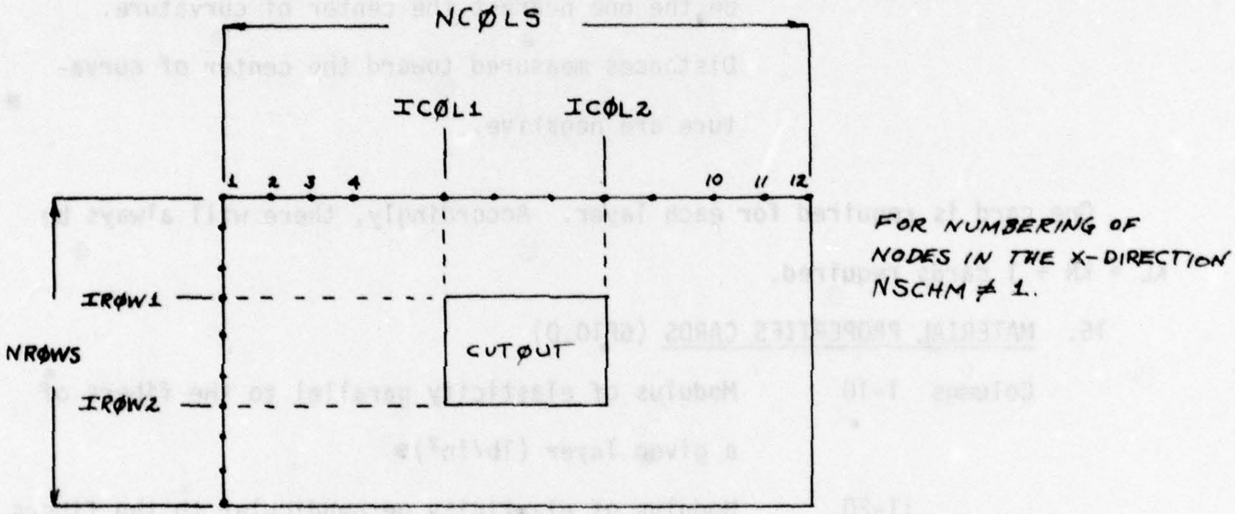
One card is required for each layer. Accordingly, there will always be KL = KN + 1 cards required.

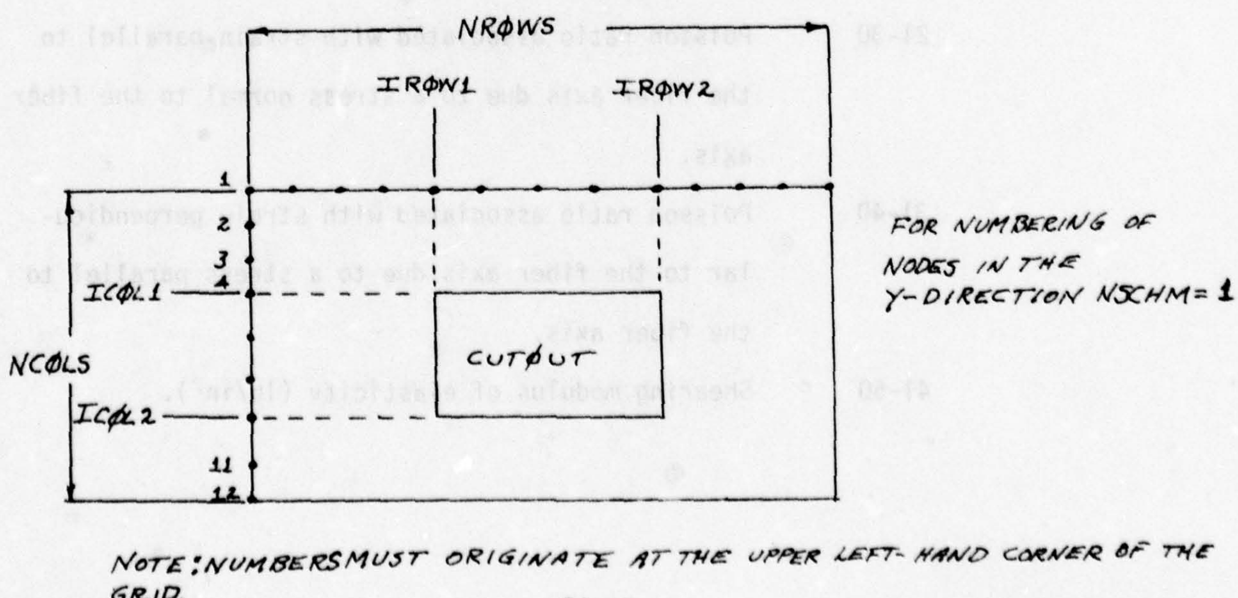
15. MATERIAL PROPERTIES CARDS (6F10.0)

Columns	1-10	Modulus of elasticity parallel to the fibers of a given layer (lb/in^2).
	11-20	Modulus of elasticity perpendicular to the fibers of a given layer (lb/in^2).
	21-30	Poisson ratio associated with strain parallel to the fiber axis due to a stress normal to the fiber axis.
	31-40	Poisson ratio associated with strain perpendicular to the fiber axis due to a stress parallel to the fiber axis.
	41-50	Shearing modulus of elasticitv (lb/in^2).

51-60 Angle between the fiber direction and a generator of the panel. This angle is positive whenever the fiber direction is situated in a clockwise orientation relative to the positive x-direction.

One card is required for each layer of the panel. Isotropic materials can be included by specifying a single layer.





NOTE: NUMBERS MUST ORIGINATE AT THE UPPER LEFT-HAND CORNER OF THE GRID.

OUTPUT INFORMATION

Information printed by the program is described in this section.

1. Problem identification.
2. Radius of curvature of the panel and the convergence critierion for displacements.
3. Global nodal incidences for each area element.
4. Semi-bandwidth associated with the finite difference mesh selected. (Maximum value of LBAND + 1 is 82).
5. x- and y-coordinates of each nodal point.
6. Applied load intensity for each element at each load level.
7. Total number of layers comprising the panel thickness.
8. Distance of the surface nearest the center of curvature for each layer from a selected reference surface.
9. Material properties for each layer. (Elastic modulii parallel and perpendicular to the fibers, shearing modulus of elasticity, and two Poisson ratios.)
10. Element material property matrix ELN (6,6):

$$[ELN] = \begin{bmatrix} [A] & [B] \\ [B] & [D] \end{bmatrix}$$

11. Load level and iteration number.
12. Displacements W, U, V for each nodal point of the finite-difference grid for each load level.

13. Normalized displacement

$$||W|| = \Sigma W_i^2$$

14. Determinant of the derivative matrix at the beginning of each load-level and at those iterations for which the derivative matrix has been up-dated.

15. A message is printed if convergence at any load-level fails to be achieved after 8 iterations. The analyst must analyze the load-deflection curve for a specific problem to determine if bifurcation or limit point buckling has occurred. Bifurcation buckling is detected by plotting the determinant vs load diagram. Limit point buckling is detected by a local maximum on the load-deflection curve.

16. Run number.

17. Total number of blocks of equations.

PROGRAM CAPACITY: The program with its present dimension statements can handle a maximum of 244 area-elements, 240 finite-difference grid points, and an equation system of maximum bandwidth equal to 81. To increase the program capacity changes in array dimensions listed below must be made.

1.	X (NUMNP)	NUMNP = Total number of grid points
2.	Y (NUMNP)	
3.	ID (BP)	BP = 6 * (Number of boundary points)
4.	NODE (NEL, 9)	NEL = Total number of area-elements
5.	ITYPE (NEL)	
6.	PLOAD (NEL, 3)	
7.	PINCR (NEL, 3)	
8.	PINCO (NEL, 3)	
9.	IBC (NEL, 3)	
10.	MAXA (MI)	MI = LBAND + NEQB - 1
11.	MAXB (MI)	
12.	AK (3*NEQB, NEQB)	NEQB = Number of equations per block
13.	NODES (NEL)	(NEQB should be greater than or equal to LBAND. Change NEQB according to the size
14.	CC (NEQB)	of the bandwidth of the system.)
15.	P1 (NAV)	NAV = NEQB* (LBAND + 1)
16.	P2 (NAV)	
17.	D (NAV)	
18.	SC (NBLOCK)	NBLOCK = Total number of blocks of equations
19.	Q (NAR)	NAR = NEQB * NBLOCK + LBAND
20.	P6 (MI)	

Procedure Used to Generate The Load-deflection Curve in Segments

Occasionally, it is difficult to generate the entire load-deflection curve in a single computer run. The present program has the capability of generating the load-deflection curve in separate segments. This procedure uses two subroutines, STIME and TTIME, that are available in the scientific subroutine library at the Clemson University Computer Center. The user is therefore advised to refer to the manuals on scientific subroutines for corresponding subroutines.

Subroutine STIME (INDEX)

This routine initializes the computer time at the beginning of the program.

Subroutine TTIME (INDEX)

This routine calculates the exact computer time involved from the initialization point and stores it in INDEX. There on every time this routine is called it adds to the previous value of Index the current increment of time and stores it in Index. Index is an integer which is expressed as units of 26.01 microseconds of the computer time.

ITIME - Maximum value of the computer time allocated by the user for a particular computer run. It is an integer value expressed as units of 26.01 microseconds. If INDEX is greater than the ITIME, the current variables which are needed to generate the next segment are stored on the tape NTAPE and the program is terminated. Subroutine TTIME is called only after each load-level and so the value of ITIME should be less than the time specified on the job control card.

FAST STORAGE ALLOCATION

The total program storage requirement, including program instructions and array requirements, is 272K bytes. The following information indicates the storage requirements for the various arrays that appear in the program.

	Fast storage requirement (bytes)
1. Maximum number of finite-difference grid points is 240. Accordingly,	
x(240)	1K
y(240)	1K
2. Maximum semi-bandwidth plus one column for a load vector is 81. Accordingly, fast storage allocated to the generation of the stiffness matrix is:	
AK(246,82)	81K
3. The entire displacement vector is required to be in fast storage. Also working space is required to process displacements associated with a block of equations. Thus,	
Q(850)	
CC(100).	4K
4. Arrays whose dimensions are permanent.	36K

5. Maximum number of area-elements is 244.

Accordingly, space is required for:

NODE(244,9)

PLOAD(244,3)

PINCR(244,3) 23K

IBC(244,3)

ITYPE(244)

PINCO(244,3)

NODES(244)

6. An auxilliary matrix for processing boundary conditions is required. Thus,

ID(400) 2K

7. Maximum number of blocks of equations is 10.

8. Subroutines MULTOC and SESOL require fast storage working space of:

MAXA(170)

MAXB(170)

P6(100)

The working arrays P1, P2 and D are equivalenced to the matrix AK and so extra storage space is not needed. 2K

9. Working area in ATBA:

BB(12,27) 2K

10. Storage for material properties and

material stiffness matrix

Q(10,3,3)	
A(3,3)	
B(3,3)	1K
D(3,3)	

Total Array Storage Requirements 153K

APPENDIX D

SUBROUTINE DESCRIPTIONS

Appendix D contains brief descriptions of the overall purposes of the subroutines used by the main program with detailed explanations of the various parameters that appear in their argument lists.

1. SUBROUTINE ATBA (A,B,C)

Description. Subroutine ATBA computes the triple matrix product:

$$[C] = [A]^T[B][A]$$

Arguments. The parameters appearing in the argument list are defined as follows:

A	Matrix of dimensions 27 x 27
B	Matrix of dimensions 12 x 27
C	Matrix of dimensions 27 x 27

The matrix [A] in the triple matrix product required at all points of the present program has a special form for which two thirds of its elements are zero. Consequently, special coding was devised to take advantage of the special character of [A].

2. SUBROUTINE BCS (A,B,MAXA,NAV,NEQB, LBAND,MI,KEX,NBLOCK,NC,ND,MAXB)

Description. Subroutine BCS establishes support conditions along exterior and interior edges of the panel. Permissible edge conditions are either zero force or zero displacement for each degree of freedom at each boundary point.

Arguments. Parameters appearing in the subroutine argument list are defined as follows:

A	Vector of dimension NAV
B	Vector of dimension NAV
NAV	NEQB x (LBAND + 1)
NEQB	Number of equations in a block
MAXA	Vector of dimension MI
MAXB	Vector of dimension MI
MI	NEQB + LBAND - 1
LBAND	Maximum semi-bandwidth
NBLOCK	Number of blocks of equations
KEX	Index that determines when refactorization of the coefficienty matrix is required. If KEX = 1 no refactorization is required; if KEX = 2 refactorization is required.

NC	Tape number on which the derivative of the stiffness matrix and load vector are written before boundary conditions have been applied.
ND	Tape number on which the derivative of the stiffness matrix and the load vector are written after boundary conditions have been applied.
ID	Vector whose elements are the numerical labels associated with equations to which displacement boundary conditions are to be applied. This vector is determined in Subroutine BCID.
IK	Total number of equations to which displacement boundary conditions are to be applied. This quantity is determined in the Subroutine BCID.

3. SUBROUTINE BCID (NEL,NEQ,NBLOCK,NEQB,KBLOCK,NN)

Description. Subroutine BCID generates a vector, ID, whose elements are the numerical labels associated with system equations to which displacement boundary conditions are to be applied. BCID also determines, NN, the last nonzero diagonal element of the last block of equations.

Arguments. Parameters appearing in the subroutine argument list are defined as follows:

NEL	Total number of area-elements.
NEQ	Total number of degrees of freedom of the system.
NBLOCK	Number of blocks of equations.
NEQB	Number of equations in a block.
KBLOCK	Number of the block that contains the last nonzero diagonal element.
NN	Label number of the equation that corresponds to the last nonzero diagonal element of the stiffness matrix.

4. SUBROUTINE COEFF (NFLAT)

Description. This subroutine calculates elements of the matrices PA2(12,12), PA3(12,12,12), and PA4(12,12,12,12)*. These matrices depend only upon the material property matrix ELN(6,6) and the radius of curvature R.

Arguments. NFLAT — Index that designates either a flat plate or a curved panel.

PA2(12,12) Matrix of coefficients associated with quadratic displacement terms in the energy expression.

PA3(12,12,12) Matrix of coefficients associated with cubic displacement terms in the energy expression.

PA4(12,12,12,12)* Matrix of coefficients associated with quartic displacement terms in the energy expression.

*This array has been replaced by 15 two-dimensional arrays.

5. SUBROUTINE ELMNT (M,NDQK)

Description. This subroutine calculates the stiffness matrix QK(27,27), its derivative DQK(27,27), and the element load vector RE(27) for each area element. The basic equations are as follows:

Stiffness matrix;

QK(I,J) = (CA2(I,J) + BN1(I,J)/2 + BN2(I,J)/3)*AREA

Derivative matrix;

DQK(I,J) = (CA2(I,J) + BN1(I,J) + BN2(I,J))*AREA

The matrices appearing in these expressions are calcuated from the equations:

$$[CA2] = \begin{cases} [C]^T[PA2][C] & \text{- without imperfections} \\ [C]^T[PA2T]\,[C] & \text{- with imperfections} \end{cases}$$

$$[BN1] = [C]^T[AN1]\,[C]$$

$$[AN1] = \begin{cases} 3PA3_{ijk}\tilde{d}_k & \text{- without imperfections} \\ 3PA3T_{ijk}\tilde{d}_k & \text{- with imperfections} \end{cases}$$

$$[BN2] = [C]^T[AN2][C]$$

$$[AN2] = 6\ PA4^*_{ijk\ell}\tilde{d}_k\tilde{d}_\ell \quad \text{- with or without imperfections}$$

Arguments. Arguments appearing in the subroutine parameter list have the following meanings:

M — Number of area elements to be processed.
NDQK — Tape number on which the elements of the derivative of the element stiffness matrix are stored.

*PA4(K,J,K,L) has been replaced by 15 two-dimensional arrays.

6. SUBROUTINE ELPROP

Description. This subroutine computes the stiffness properties associated with the panel wall construction. The pertinent relations are:

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ --- \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \left[\begin{array}{c|c} [A] & [B] \\ \hline [B] & [D] \end{array}\right] \begin{bmatrix} e_x \\ e_y \\ 2e_{xy} \\ ----- \\ K_x \\ K_y \\ 2K_{xy} \end{bmatrix},$$

where

$$A_{ij} = \sum_{k=1}^{KN} \bar{Q}_{ij}{}^{(k)}(h_k - h_{k-1}),$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^{KN} \bar{Q}_{ij}{}^{(k)}(h_k^2 - h_{k-1}^2),$$

and

$$D_{ij} = \frac{1}{3} \sum_{k=1}^{KN} \bar{Q}_{ij}{}^{(k)}(h_k^3 - h_{k-1}^3),$$

INPUT.

KN	Total number of layers
H(K)	Distance of the surface of the k-th layer nearest the center of curvature from the reference surface.
E1, E2, ENU1, ENU2, G, TT	- Elastic modulii, Poisson's ratios, shear modulus, and fiber orientation, respectively. These six quantities are to be input for each layer.

OUTPUT. The material stiffness matrix is returned to the main program in the array ELN(6,6).

7. SUBROUTINE FDIFF (NEL,NUMNP,NX1,NC2,WO,CONS1,CONS2,XO,YO,IEX,IPA3T)

Description. This subroutine computes the element area, element centroid, and the finite difference formulas for the centroidal displacements and their derivatives for each type of element (interior or boundary element). The pertinent relation is:

$$\begin{bmatrix} g \\ g_{,x} \\ g_{,y} \\ g_{,xx} \\ g_{,xy} \\ g_{,yy} \end{bmatrix} = [B] \begin{bmatrix} W_1 \\ U_1 \\ V_1 \\ W_2 \\ U_2 \\ V_2 \\ \cdot \\ \cdot \\ W_9 \\ U_9 \\ V_9 \end{bmatrix}$$

Arguments. Parameters appearing on the subroutine argument list are defined as follows:

NEL	Number of area elements.
NUMNP	Number of nodal points in the finite-difference grid.
NX1	Tape number upon which the matrix [B] and the area for each element are written.
NC2	Tape number upon which the matrix [CA2] is written for each element.
WO	Amplitude of initial geometric imperfection.
CONS1	Value of $m\pi/a$, where a is the length of the panel parallel to a generator.
CONS2	Value of $n\pi/b$, where b is the width of the panel parallel to a cross sectional circle.
XO,YO	Coordinates of the corner of the panel nearest the origin of the x,y system.
IEX	Index that determines whether or not an imperfection analysis is to be performed.
IPA3T	Tape number upon which the matrix [PA3T] = [PA3] + [PA3I] is written for each area element.
B(6,9)	6x9 matrix that contains the finite-difference coefficients that express a function value or its derivative value as a linear combination of the local nodal displacements.
AREA	Element area.

<u>Notes.</u>

ITYPE(1) = 1 - Left exterior or right interior boundary element.
ITYPE(2) = 2 - Top exterior or bottom interior boundary element.
ITYPE(3) = 3 - Right exterior or left interior boundary element.
ITYPE(4) = 4 - Bottom exterior or top interior boundary element.
ITYPE(5) = 5 - Interior boundary element.
ITYPE(6) = 6 - Upper-left corner element.
ITYPE(7) = 7 - Upper-right corner element.
ITYPE(8) = 8 - Lower-left corner element.
ITYPE(9) = 9 - Lower-right corner element.

8. SUBROUTINE IBAND (LBAND,NEL)

<u>Description</u>. Subroutine IBAND determines the semi-bandwidth of the system stiffness matrix. The semi-bandwidth is calculated from the formula

LBAND = 3*(LARGE + 1),

where: LARGE = MAXO(ABS(NODE(M,9) - (NODE(M,7))

over all area elements.

LBAND	Semi-bandwidth for the system stiffness matrix.

NEL	Total number of area elements.

9\. SUBROUTINE IMCOEF (NC2,IPA3T,XC,YC,WO,CONS1,CONS2,XO,YO,NFLAT)

Description. This subroutine calculates the coefficients associated with quadratic and cubic displacement terms in the energy expression due to initial geometric imperfections. These coefficients depend only upon the material property matrix ELN(6,6), the radius of curvature R, and the initial geometric imperfections.

This subroutine also adds the contributions of PA2(12,12) to the coefficients associated with quadratic displacements due to initial imperfections PA3(12,12) and PA3T(12,12,12).

Arguments. Arguments appearing in the subroutine parameter list have the following meanings:

NC2	Tape number on which the elements of CA2 are written (i.e., linear portion of the stiffness matrix).
IPA3T	Tape number on which the elements of PA3T are written for each element.
NFLAT	Index that designates either a flat plate or a curved panel.
XC,YC	x- and y- coordinates of the centroid of an element.
WO	Amplitude of initial geometric imperfections.
CONS1,CONS2	Constants associated with the form of the imperfection.
XO,YO	x- and y- coordinates of the lower left-hand corner of the panel.

10\. SUBROUTINE MULTOC (A,B,C,D,Q,MAXA,NAV,NEQB,NAR,LBAND,NBLOCK,MI, NSTIF,KSTIF,NC,KEX)

Description. This subroutine multiples a vector x by a square matrix A. The entire vector x is assumed to be contained in fast storage, while the square matrix A is brought into core by blocks.

Arguments. Arguments appearing in the subroutine parameter list have the following meanings:

A,B,C,	Vectors of dimension NAV
C	Vector dimension NEQB
Q	Vector dimension NAR
NEQB	Number of rows in a block.
LBAND	Semi-bandwidth of the coefficient matrix.
NBLOCK	Number of equal blocks into which the coefficient matrix is partitioned.

NAV	NEQB*(LBAND+1)
NAR	NEQB*NBLOCK+LBAND
NSTIF	Tape number on which the stiffness matrix is written.
KSTIF	Tape number on which the derivative of the stiffness matrix is written.
NC	Tape number on which the result of the multiplication is written. Each file on this tape contains the derivative of the stiffness matrix as its first LBAND columns and the result of the multiplication as the LBAND+1 column for a single block.
MAXA	Vector of dimension MI
MI	NEQB+LBAND-1
KEX	Index that determines when refactorization of the coefficient matrix is required.

11. SUBROUTINE SESOL (A,B,MAXA,NEQ,MA,NV,NBLOCK,NEQB,NAV,MI,NSTIF,NRED, NL,NR,KBLOCK,NN,KEX,MAXB,DET)

Description. This subroutine solves a banded, symmetrical set of linear algebraic equations.

Arguments. Arguments appearing in the subroutine parameter list have the following meanings:

A,B	Vectors of dimensions NAV
MAXA,MAXB	Vectors of dimension MI
NEQ	Total number of equations
MA	Semi-bandwidth
NV	Number of load vectors (for this program NV=1)
NBLOCK	Number of matrix blocks into which the coefficient matrix is partitioned.
NEQB	Number of equations in a block.
NAV	NEQB*(MA+1)
MI	LBAND+NEQB-1
NSTIF	Tape number on which the original coefficient matrix is written.
NRED	Tape number on which the reduced coefficient is written.
NL	Tape number on which the solution vector is written in reverse order.
NR	Tape number of working file.
KBLOCK	Number of the block in which the last nonzero diagonal element of the stiffness matrix appears.
NN	Label number of the last equation for which the diagonal element of the stiffness matrix is nonzero

KEX	Index that determines when refactorization of the coefficient matrix is required.
DET	Determinant of the derivative of the system stiffness matrix.

12. SUBROUTINE SMALL (Q,CC,SC,NEQB,NBLOCK,LEVEL,ITER,BIG,NL,NDIS,NAR)

<u>Description</u>. Subroutine SMALL puts the solution vector in proper order and subsequently calculates the norm of the transverse displacements.

<u>Arguments</u>. Parameters appearing in the subroutine argument list are defined as follows:

Q	Displacement vector of dimension NAR
CC	Vector dimension NEQB
SC	Vector, each element of which is the maximum displacement associated with each block of equations.
NEQB	Number of equations in a block.
NBLOCK	Number of blocks of equations.
LEVEL	Label number for successive load levels.
ITER	Number of iterations of the numerical procedure at each load level.
BIG	Maximum displacement component for a converged solution at a given load level.
NL	Tape number upon which the displacements are written as they emerge from subroutine SESOL (Displacements are in reverse order).
NDIS	Tape number upon which the displacements are written as they emerge from subroutine SMALL (Displacements are in proper order).
NAR	NEQB*NBLOCK+LBAND.

13. SUBROUTINE INCDNC (NEL,NROWS,NCOLS,NB,NT,NL,NR,NSCHM,ICTOUT)

<u>Description</u>. Subroutine INCDNC establishes the correspondence between local node numbers and global node numbers.

<u>Arguments</u>. Parameters appearing in the subroutine argument list are defined as follows:

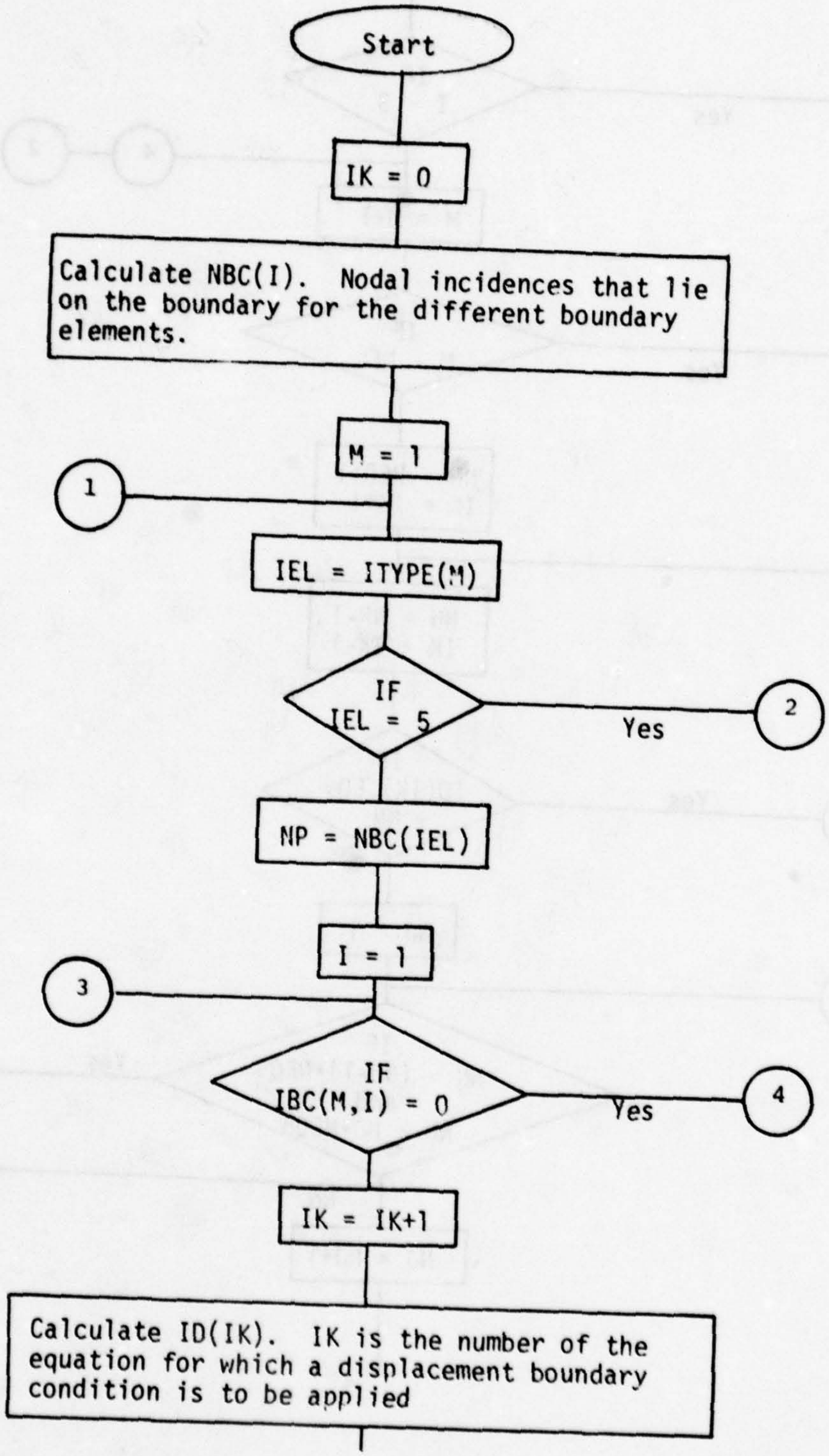
NEL	Total number of area-elements
NROWS	Number of rows of finite-difference grid points
NCOLS	Number of columns of finite-difference grid points

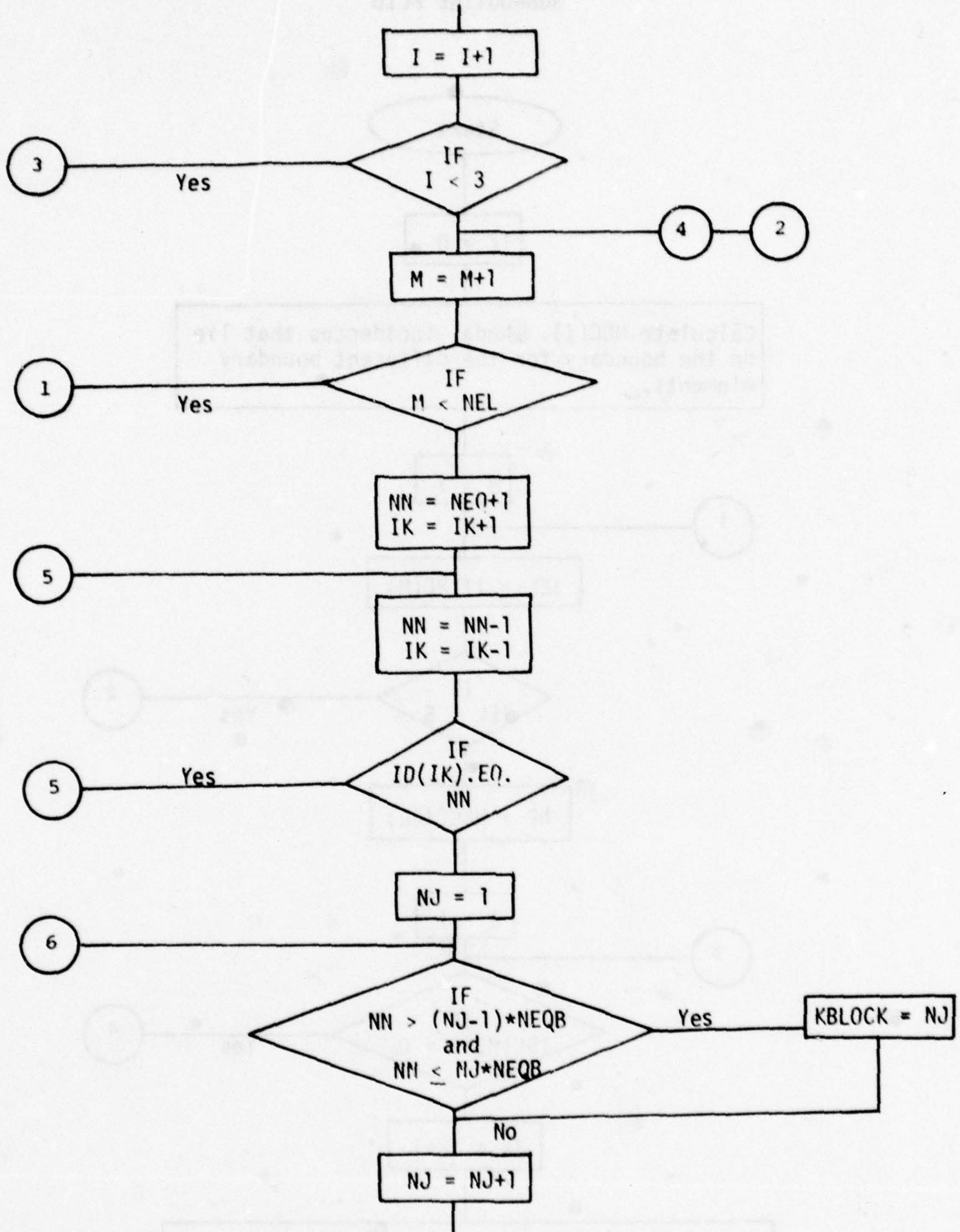
NB		Number of the row in which the side of the cutout nearest the row-baseline for the panel lies.
NT		Number of the row in which the side of the cutout furtherest from the row-baseline for the panel lies.
NL		Number of the column in which the side of the cutout nearest the column-baseline for the panel lies.
NR		Number of the column in which the side of the cutout furtherest from the column-baseline for the panel lies.
NSCHM	=1	Finite-difference nodes are numbered consecutively in the y-direction.
	≠1	Finite-difference nodes are numbered consecutively in the x-direction.
ICTOUT	=1	Panel has a cutout.
	≠1	Panel does not have a cutout.

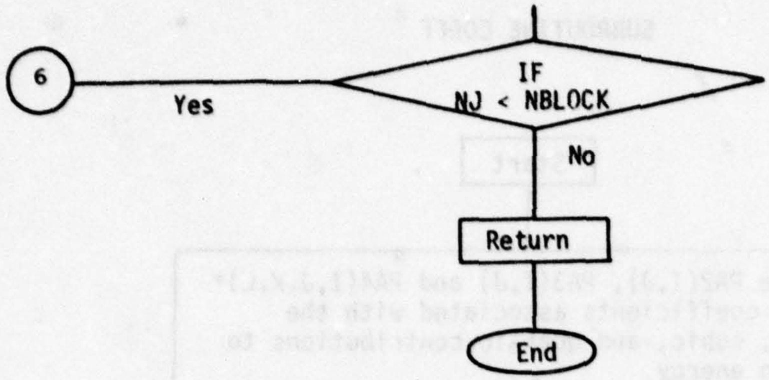
APPENDIX E

FLOW CHARTS FOR SUBROUTINES AND MAIN PROGRAM

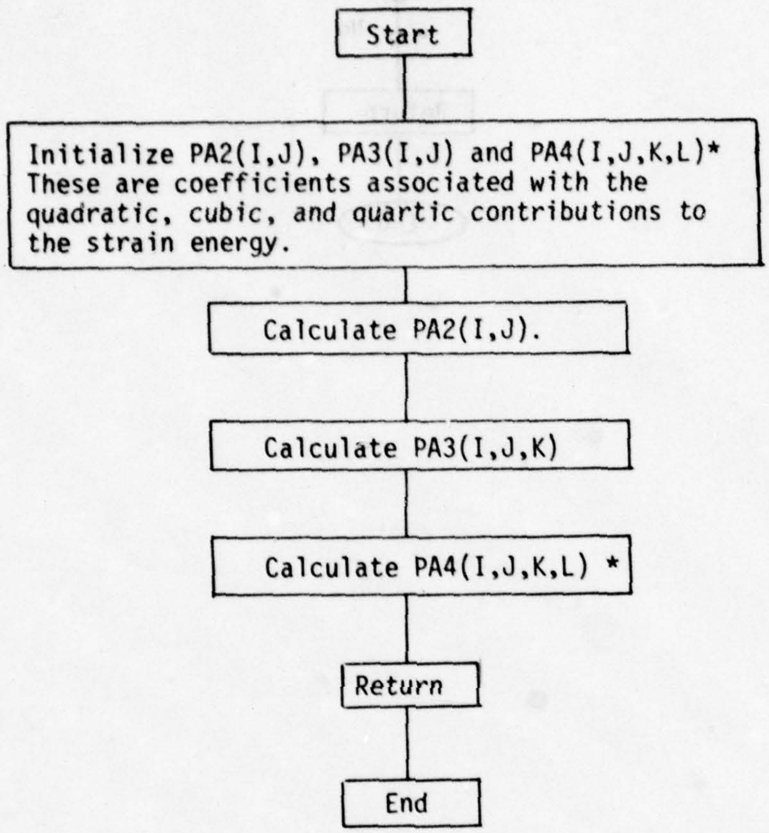
SUBROUTINE BCID




I = I+1
IF
I < 3
Yes
3
4
2
M = M+1
IF
M < NEL
Yes
1
NN = NEQ+1
IK = IK+1
5
NN = NN-1
IK = IK-1
IF
ID(IK).EQ.
NN
Yes
5
NJ = 1
6
IF
NN > (NJ-1)*NEQB
and
NN ≤ NJ*NEQB
Yes
KBLOCK = NJ
No
NJ = NJ+1

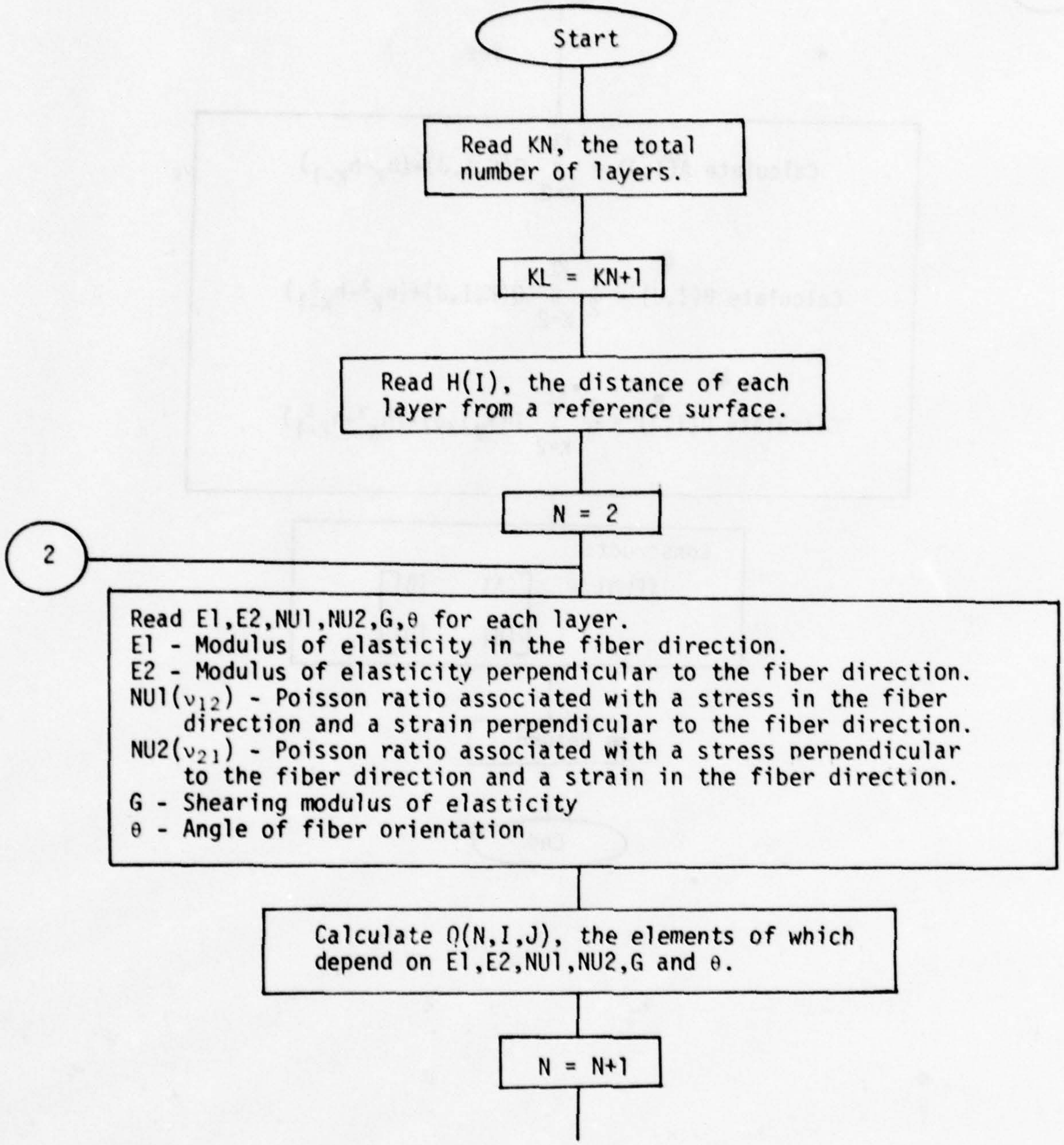

6
Yes
IF
NJ < NBLOCK
No
Return
End

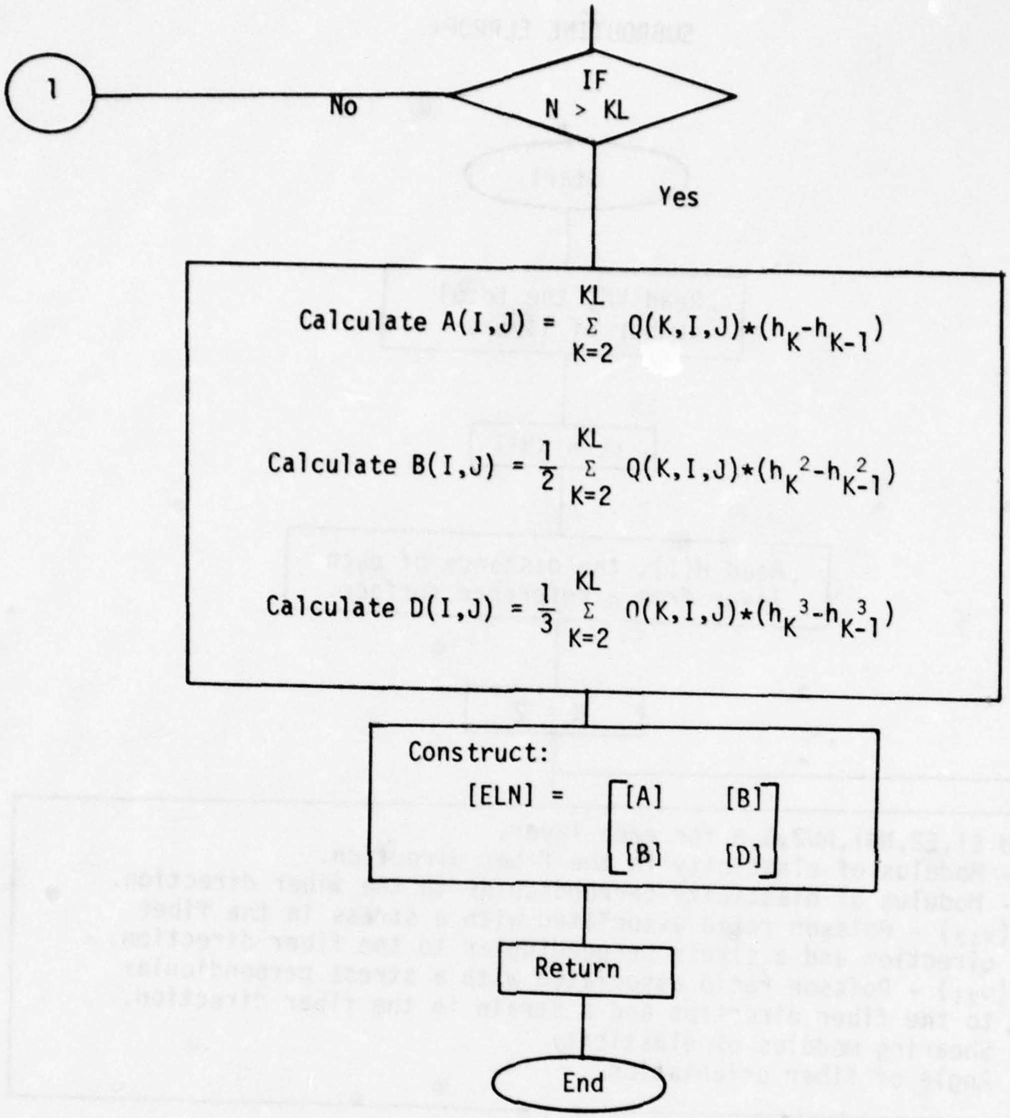
SUBROUTINE COEFF



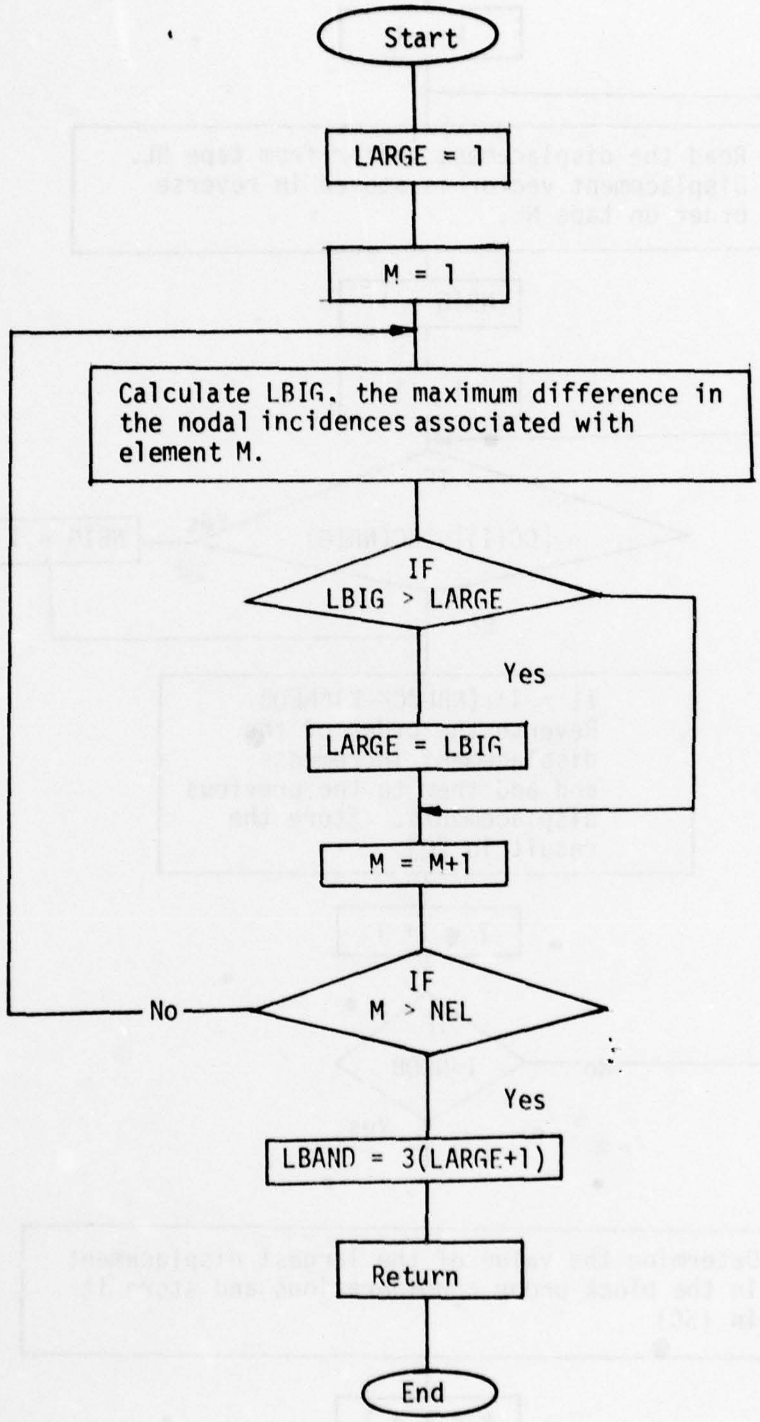
* This array has been replaced by 15 two-dimensional arrays.

SUBROUTINE ELPROP

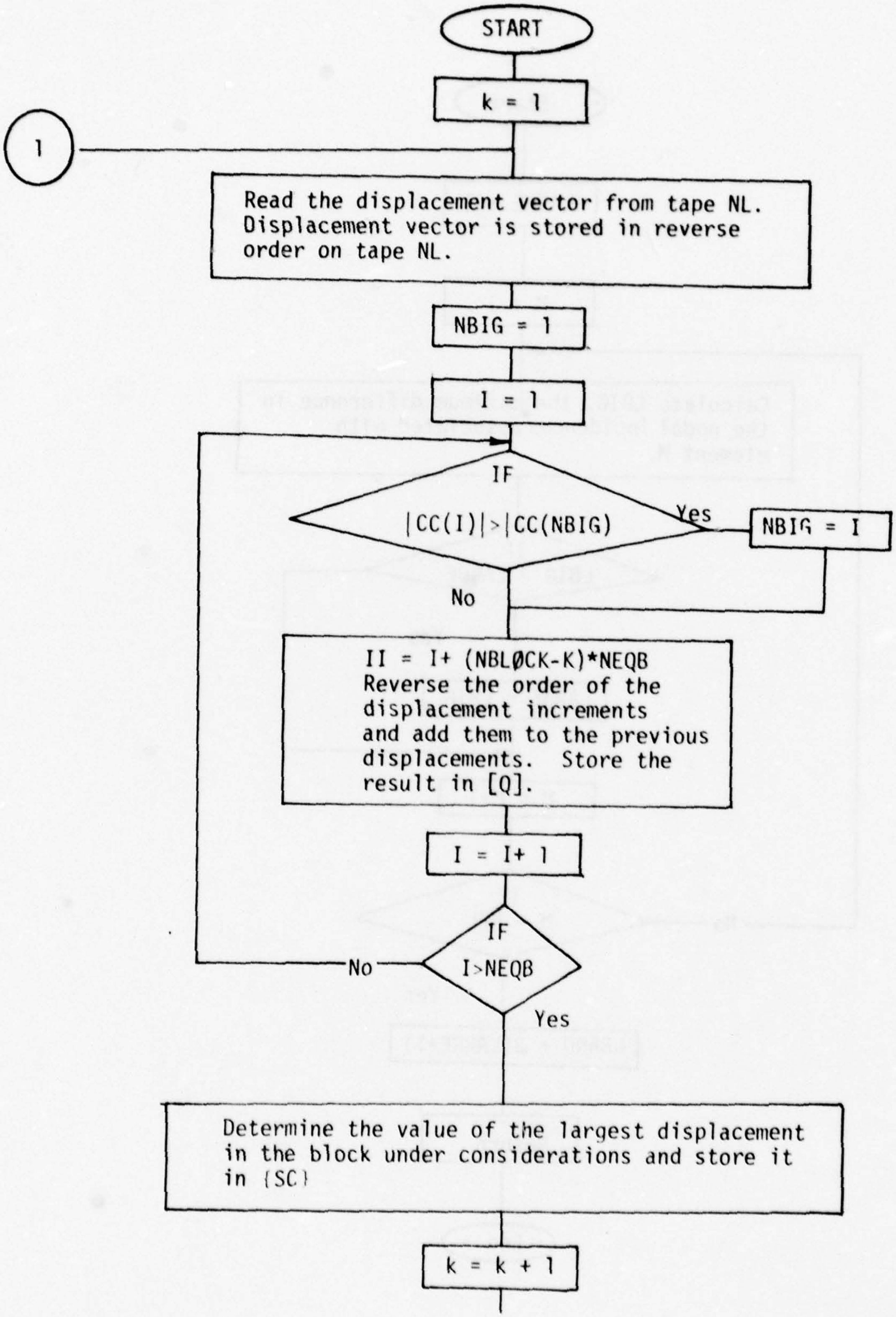


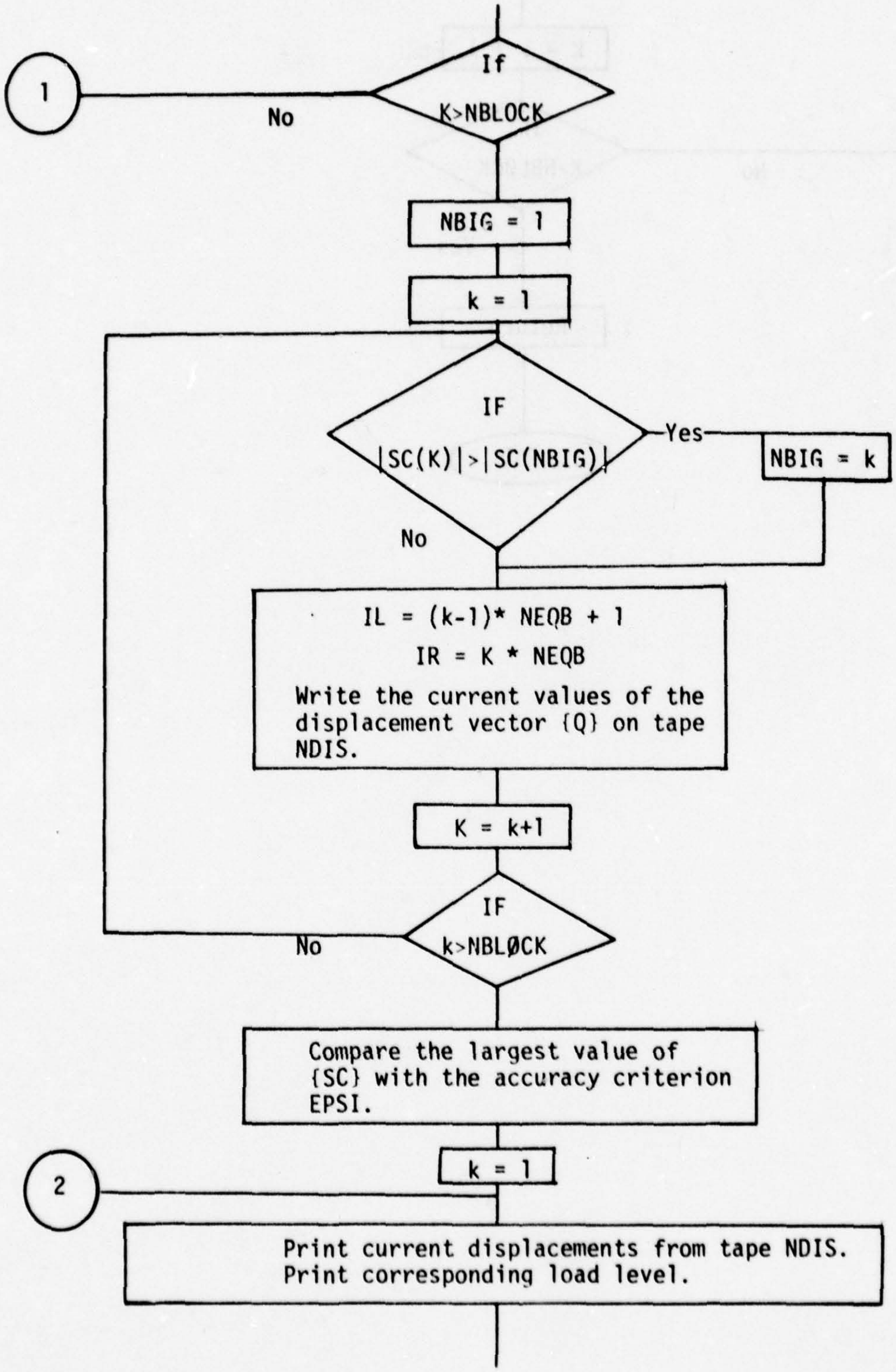

1
No
IF $N > KL$
Yes
Calculate $A(I,J) = \sum_{K=2}^{KL} Q(K,I,J)*(h_K - h_{K-1})$
Calculate $B(I,J) = \frac{1}{2} \sum_{K=2}^{KL} Q(K,I,J)*(h_K^2 - h_{K-1}^2)$
Calculate $D(I,J) = \frac{1}{3} \sum_{K=2}^{KL} Q(K,I,J)*(h_K^3 - h_{K-1}^3)$
Construct:
$[ELN] = \begin{bmatrix} [A] & [B] \\ [B] & [D] \end{bmatrix}$
Return
End

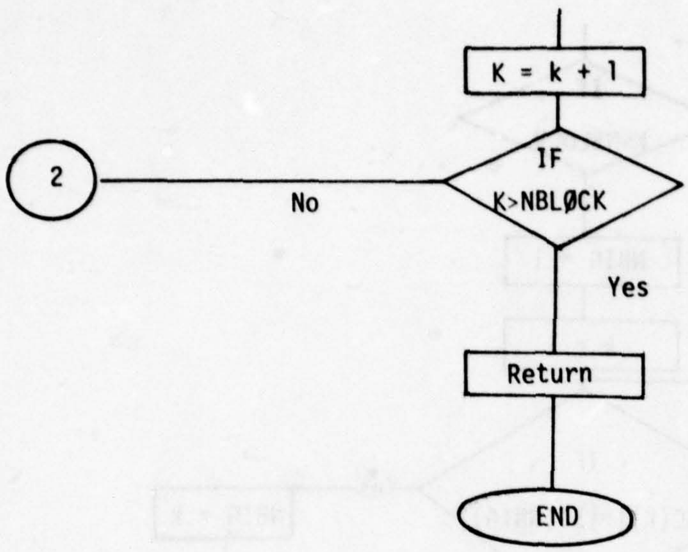
SUBROUTINE IBAND



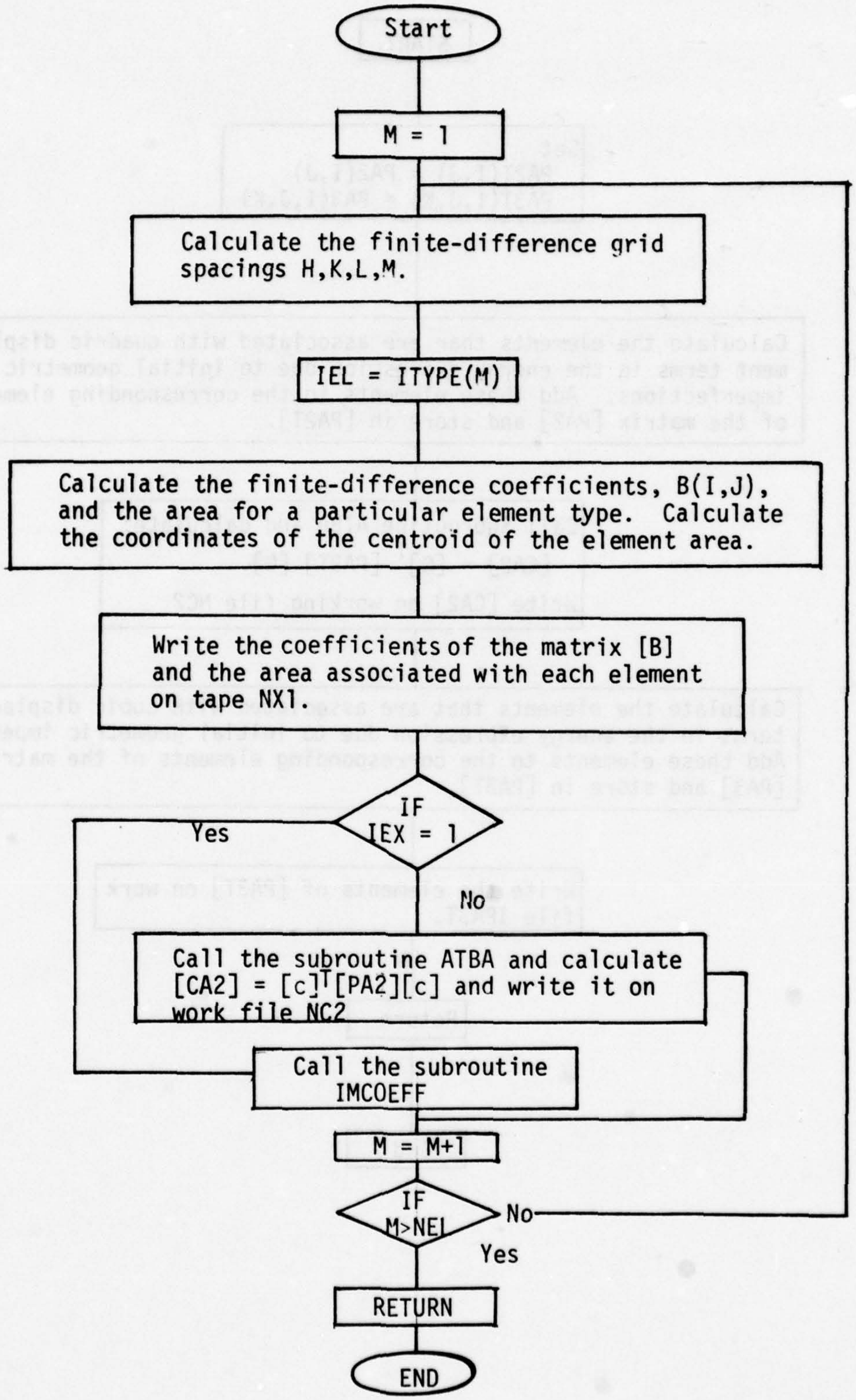
SUBROUTINE SMALL




1
If
K>NBLOCK
No
NBIG = 1
k = 1
IF
|SC(K)|>|SC(NBIG)|
Yes
NBIG = k
No
IL = (k-1)* NEQB + 1
IR = K * NEQB
Write the current values of the displacement vector {Q} on tape NDIS.
K = k+1
IF
k>NBLØCK
No
Compare the largest value of {SC} with the accuracy criterion EPSI.
k = 1
2
Print current displacements from tape NDIS.
Print corresponding load level.


K = k + 1
IF
K>NBLØCK
2
No
Yes
Return
END

SUBROUTINE FDIFF



SUBROUTINE IMCOEF

START

Set
PA2T(I,J) = PA2(I,J)
PA3T(I,J,K) = PA3(I,J,K)

Calculate the elements thar are associated with quadric displacement terms in the energy expression due to initial geometric imperfections. Add these elements to the corresponding elements of the matrix [PA2] and store in [PA2T].

Call Subroutine ATBA and calculate:
$[CA2] = [C]^T [PA2T] [C]$.
Write [CA2] on working file NC2.

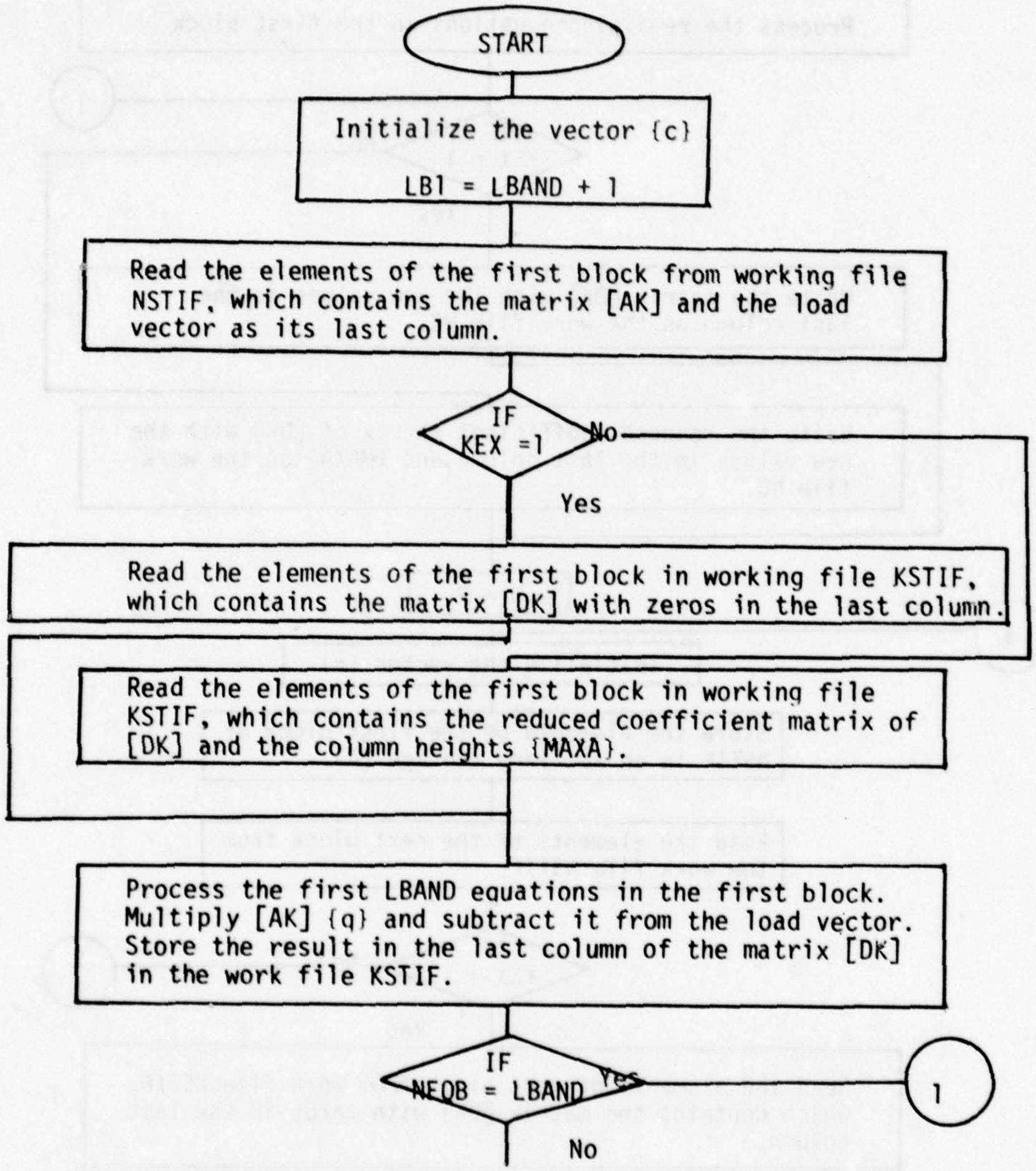
Calculate the elements that are associated with cubic displacement terms in the energy expression due to initial geometric imperfections. Add these elements to the corresponding elements of the matrix [PA3] and store in [PA3T].

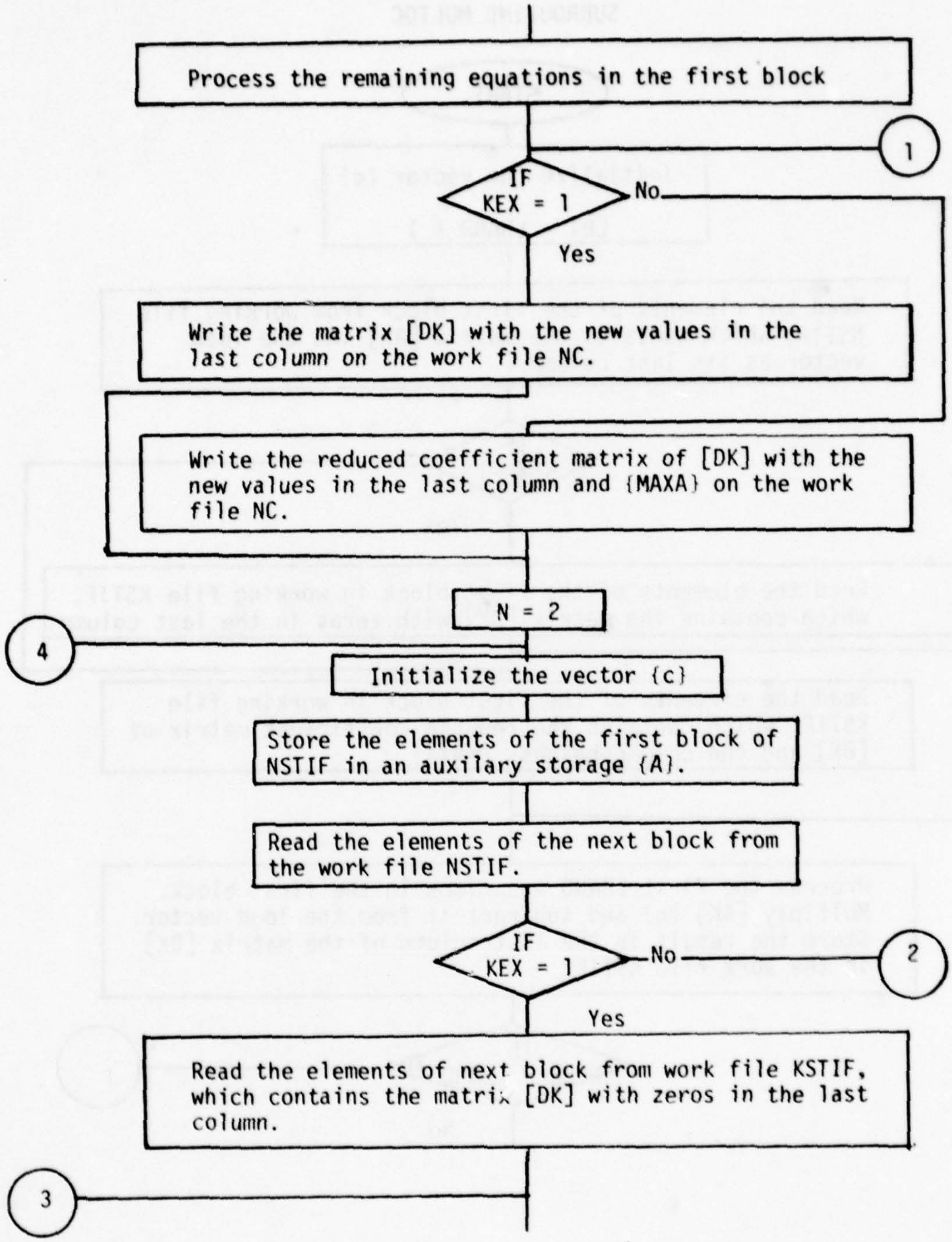
Write the elements of [PA3T] on work file IPA3T.

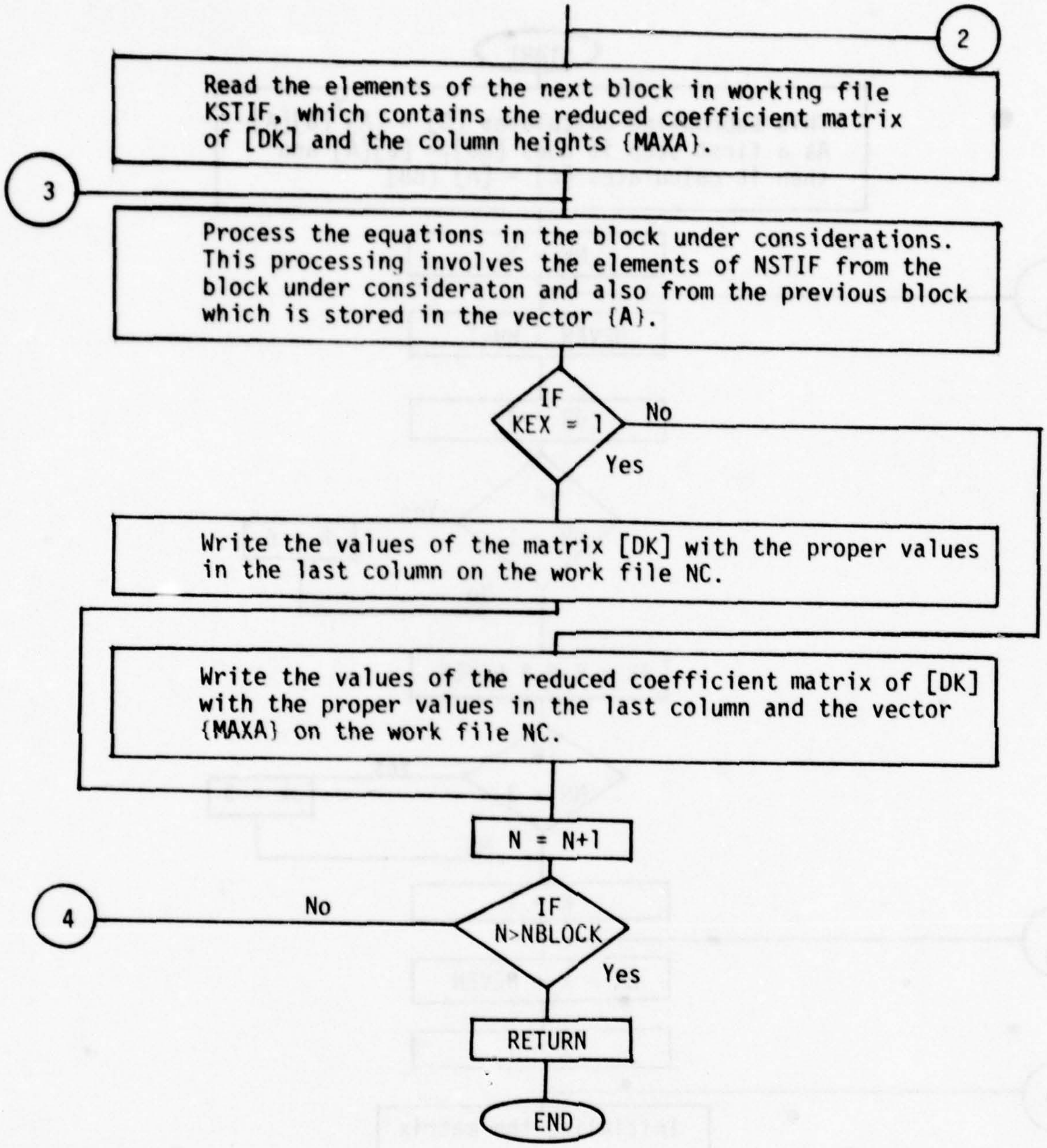
Return

End

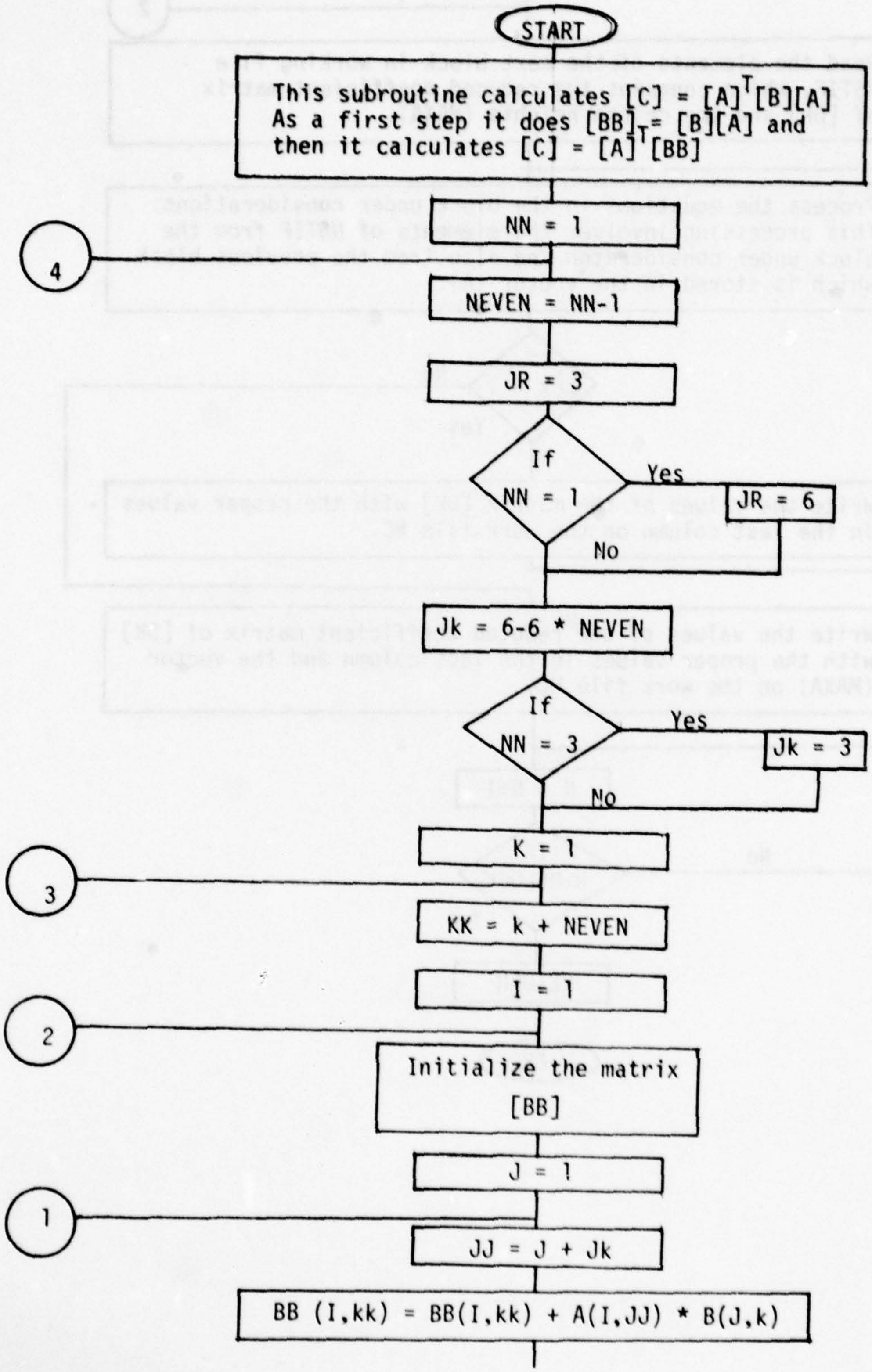
SUBROUTINE MULTOC

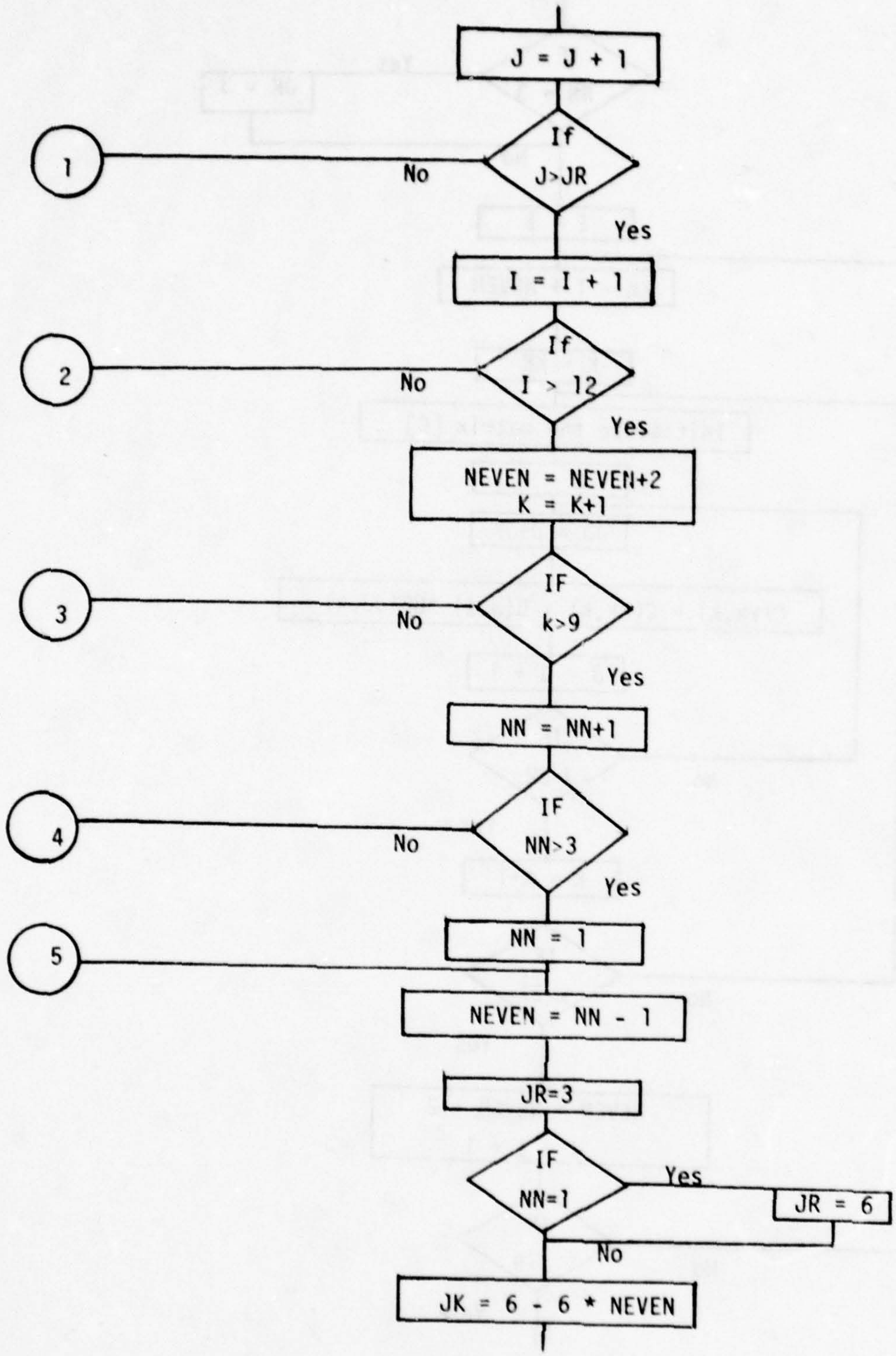


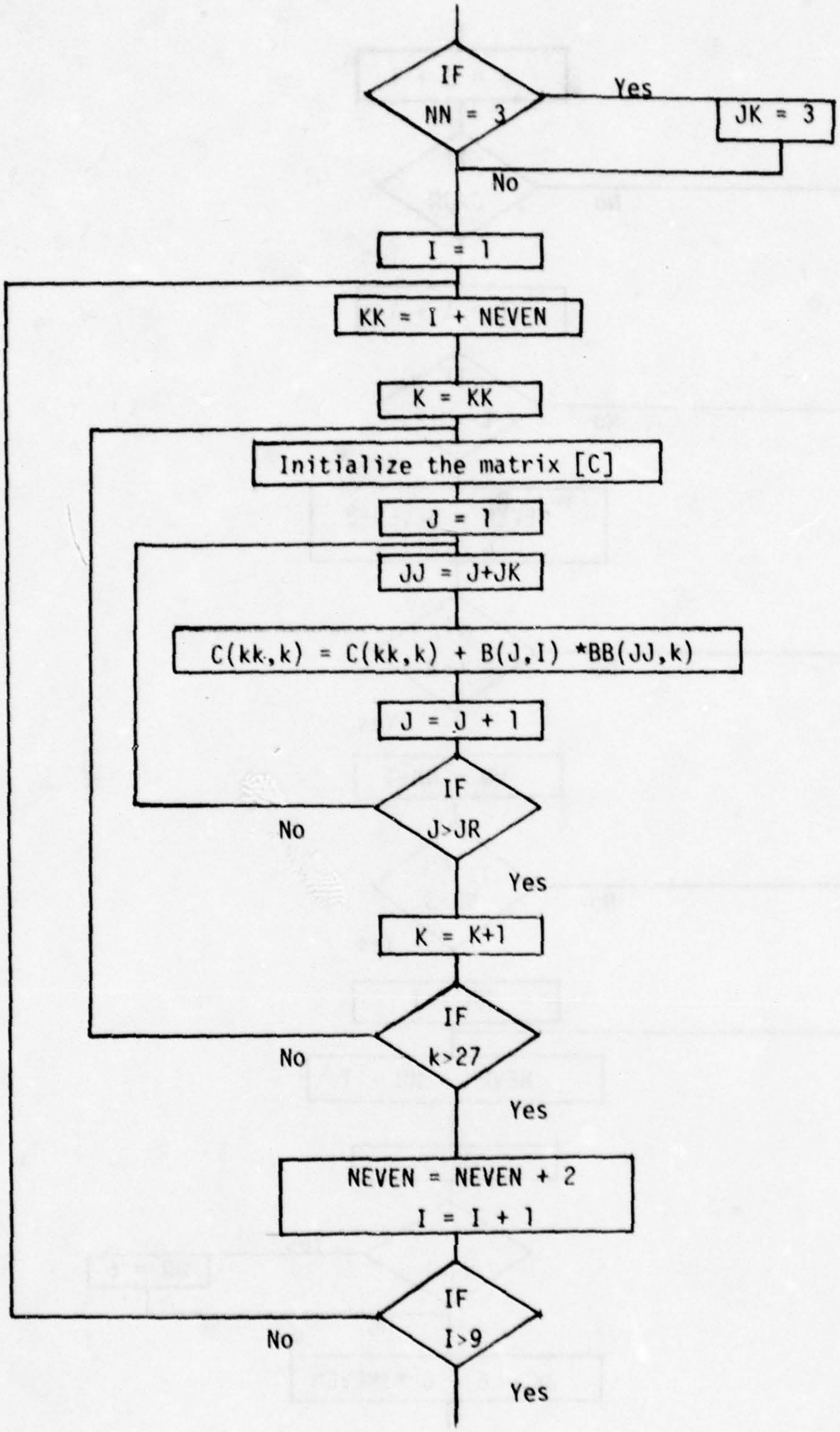

Process the remaining equations in the first block
1
IF KEX = 1
No
Yes
Write the matrix [DK] with the new values in the last column on the work file NC.
Write the reduced coefficient matrix of [DK] with the new values in the last column and {MAXA} on the work file NC.
N = 2
4
Initialize the vector {c}
Store the elements of the first block of NSTIF in an auxilary storage {A}.
Read the elements of the next block from the work file NSTIF.
IF KEX = 1
No
2
Yes
Read the elements of next block from work file KSTIF, which contains the matrix [DK] with zeros in the last column.
3

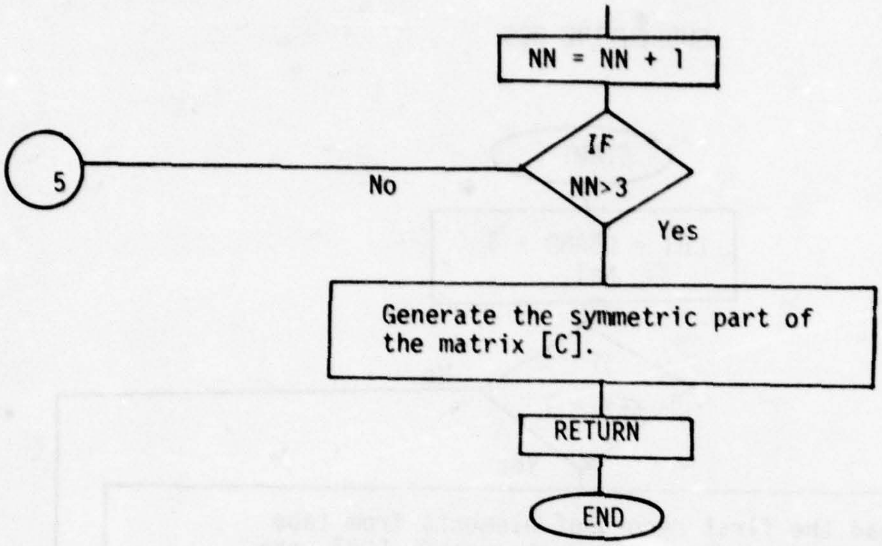

2
Read the elements of the next block in working file KSTIF, which contains the reduced coefficient matrix of [DK] and the column heights {MAXA}.
3
Process the equations in the block under considerations. This processing involves the elements of NSTIF from the block under consideraton and also from the previous block which is stored in the vector {A}.
IF KEX = 1
No
Yes
Write the values of the matrix [DK] with the proper values in the last column on the work file NC.
Write the values of the reduced coefficient matrix of [DK] with the proper values in the last column and the vector {MAXA} on the work file NC.
N = N+1
4
No
IF N>NBLOCK
Yes
RETURN
END

SUBRØUTINE ATBA

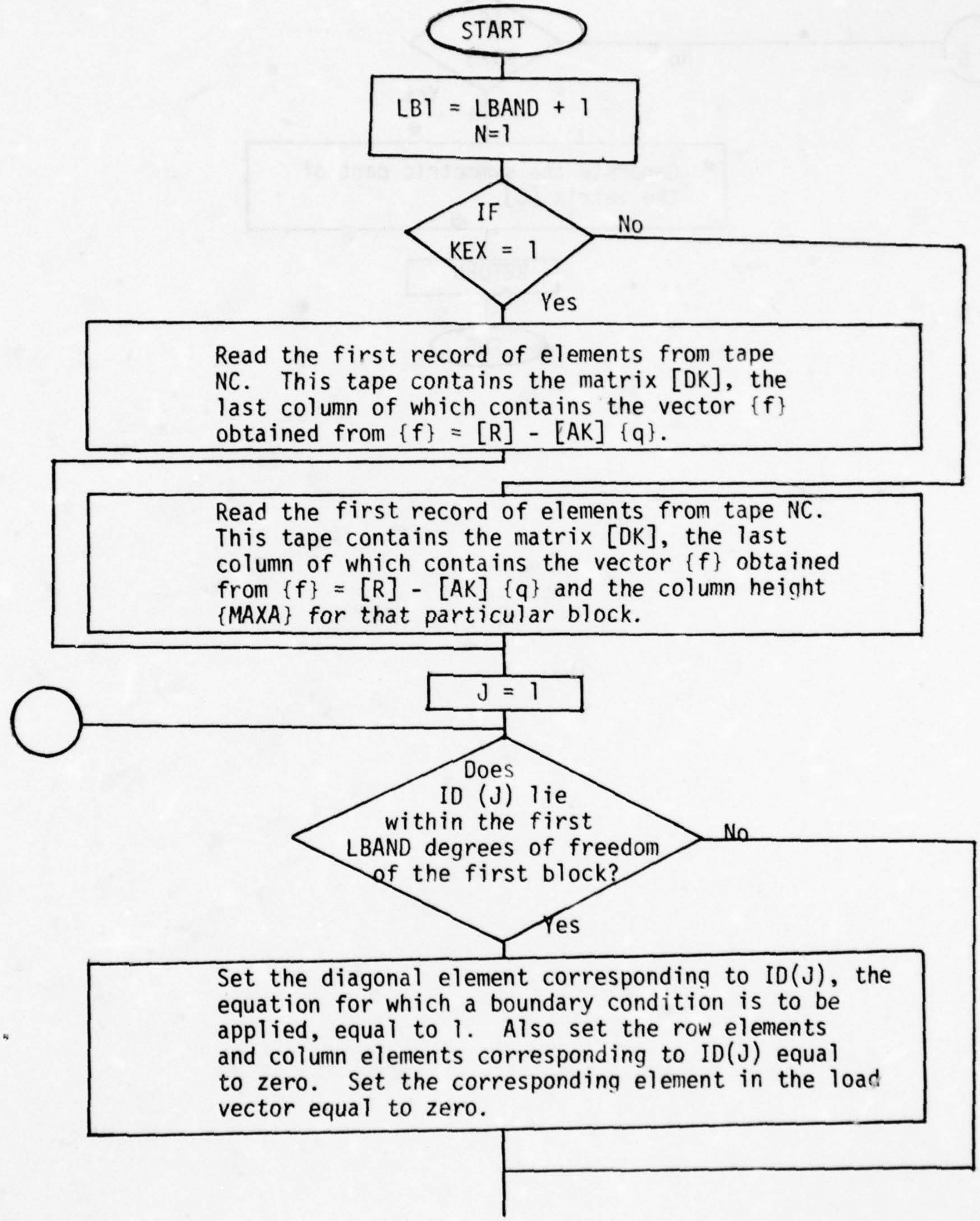


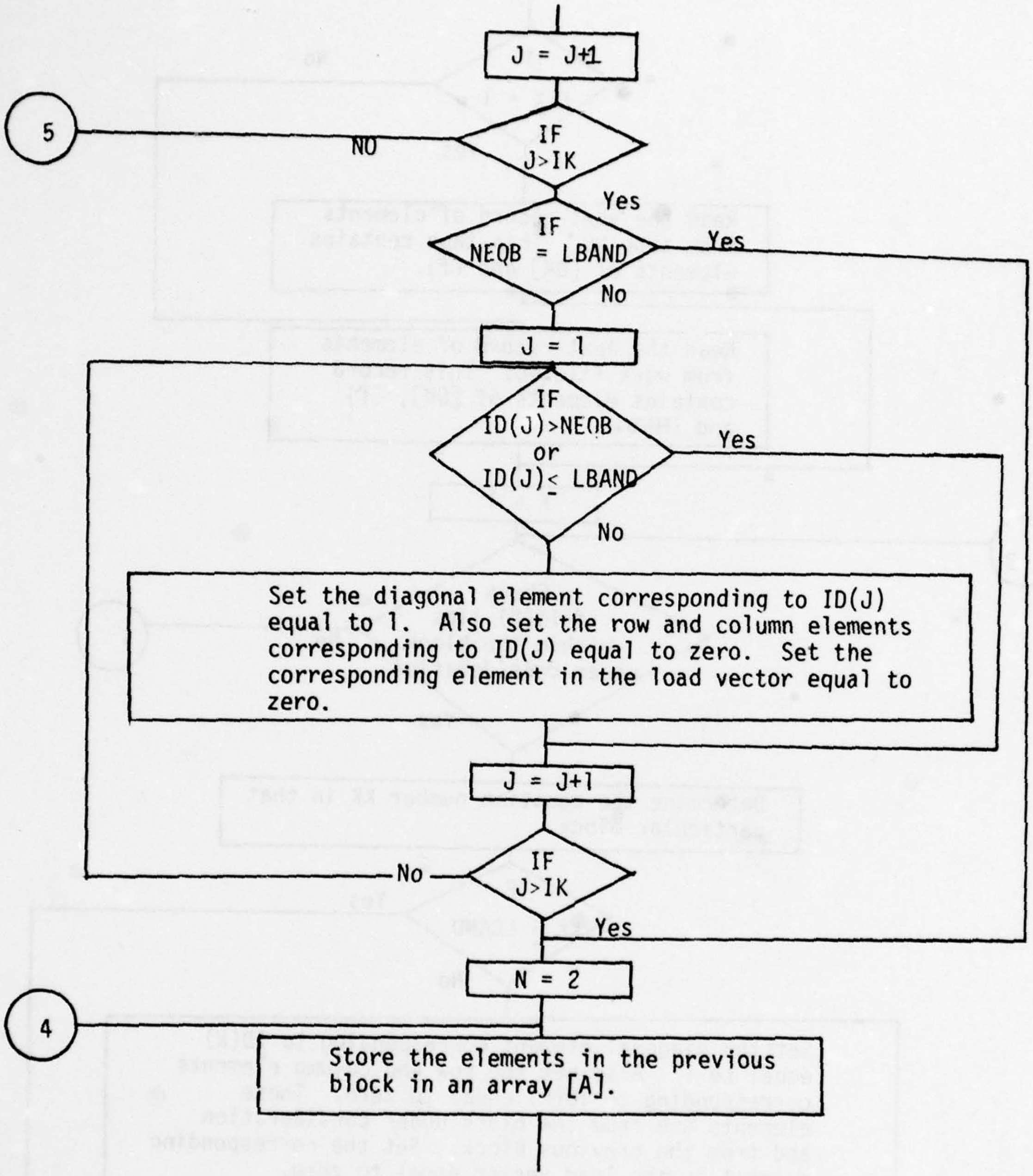

J = J + 1
If
J>JR
No
Yes
1
I = I + 1
If
I > 12
No
Yes
2
NEVEN = NEVEN+2
K = K+1
IF
k>9
No
Yes
3
NN = NN+1
IF
NN>3
No
Yes
4
NN = 1
5
NEVEN = NN - 1
JR=3
IF
NN=1
Yes
JR = 6
No
JK = 6 - 6 * NEVEN

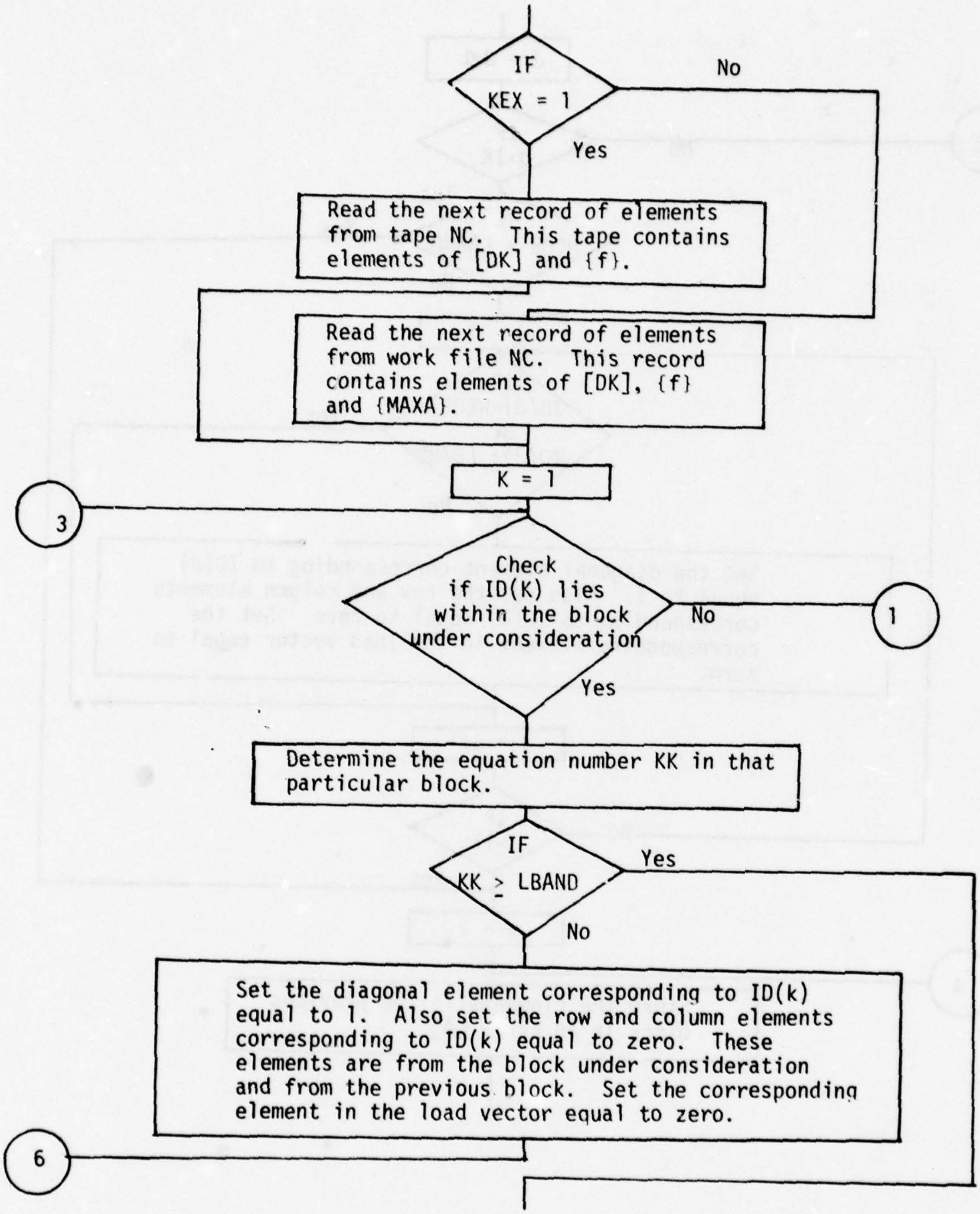

IF
NN = 3
Yes
JK = 3
No
I = 1
KK = I + NEVEN
K = KK
Initialize the matrix [C]
J = 1
JJ = J+JK
C(kk,k) = C(kk,k) + B(J,I) *BB(JJ,k)
J = J + 1
IF
J>JR
No
Yes
K = K+1
IF
k>27
No
Yes
NEVEN = NEVEN + 2
I = I + 1
IF
I>9
No
Yes

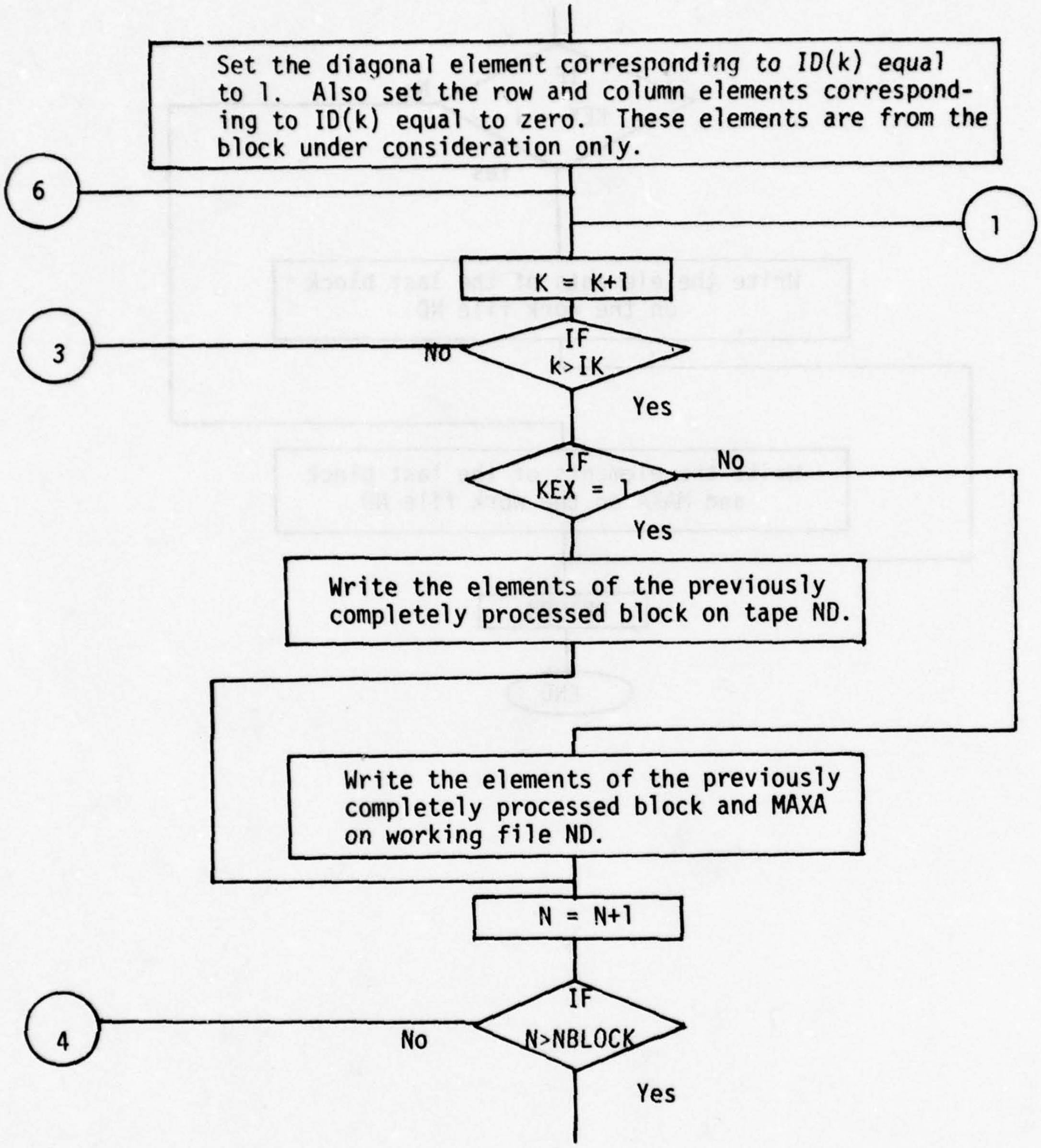

NN = NN + 1
IF
NN>3
5
No
Yes
Generate the symmetric part of the matrix [C].
RETURN
END

SUBRØUTINE BCS

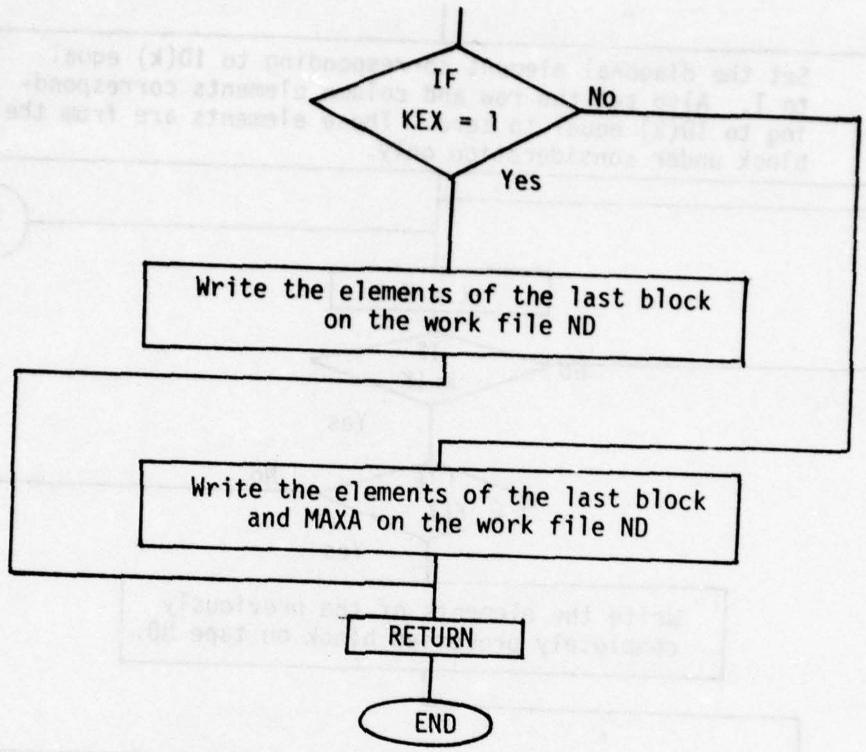



J = J+1
5
NO
IF J>IK
Yes
IF NEQB = LBAND
Yes
No
J = 1
IF ID(J)>NEQB or ID(J)≤ LBAND
Yes
No
Set the diagonal element corresponding to ID(J) equal to 1. Also set the row and column elements corresponding to ID(J) equal to zero. Set the corresponding element in the load vector equal to zero.
J = J+1
No
IF J>IK
Yes
N = 2
4
Store the elements in the previous block in an array [A].

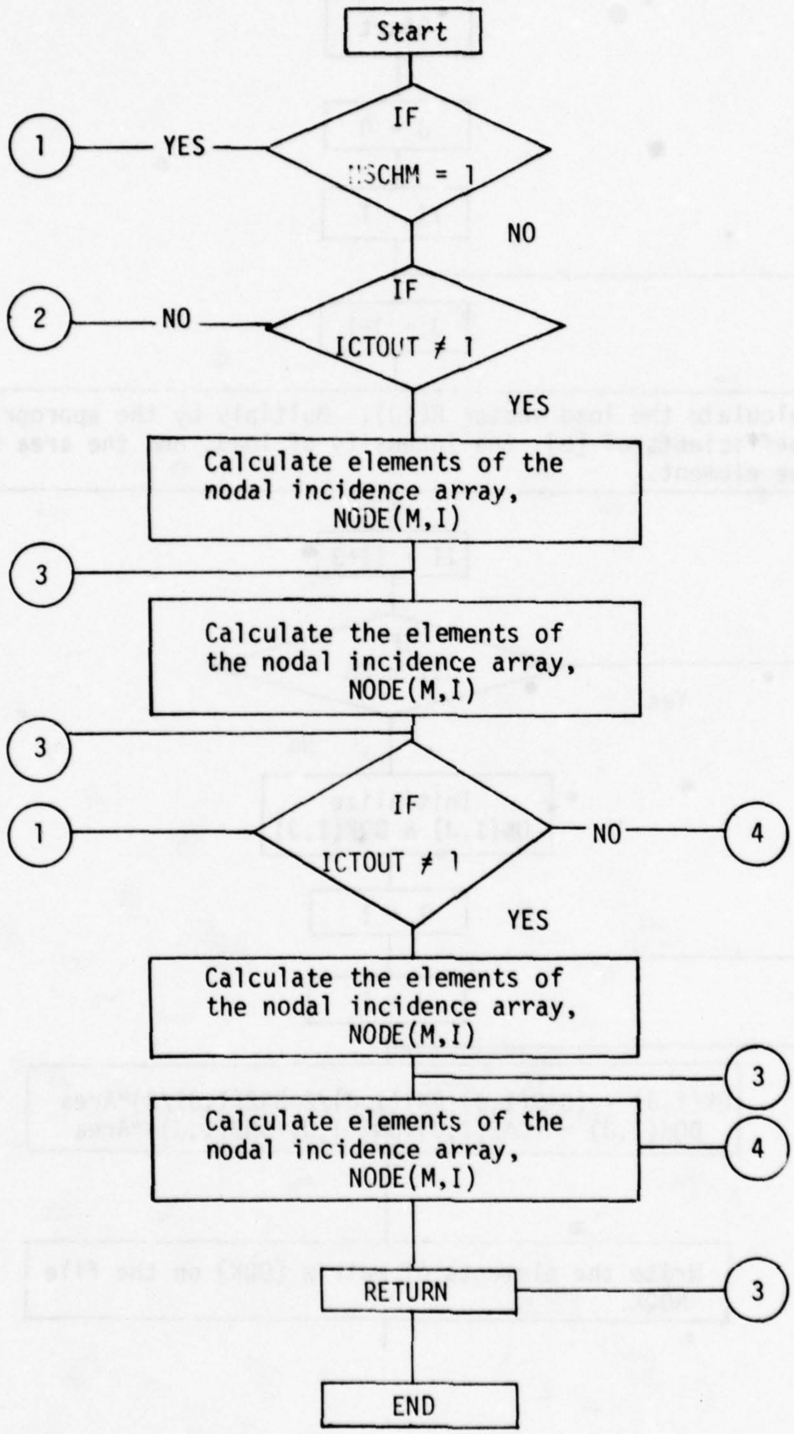

IF
KEX = 1
No
Yes
Read the next record of elements from tape NC. This tape contains elements of [DK] and {f}.
Read the next record of elements from work file NC. This record contains elements of [DK], {f} and {MAXA}.
K = 1
3
Check if ID(K) lies within the block under consideration
No
1
Yes
Determine the equation number KK in that particular block.
IF
KK ≥ LBAND
Yes
No
Set the diagonal element corresponding to ID(k) equal to 1. Also set the row and column elements corresponding to ID(k) equal to zero. These elements are from the block under consideration and from the previous block. Set the corresponding element in the load vector equal to zero.
6



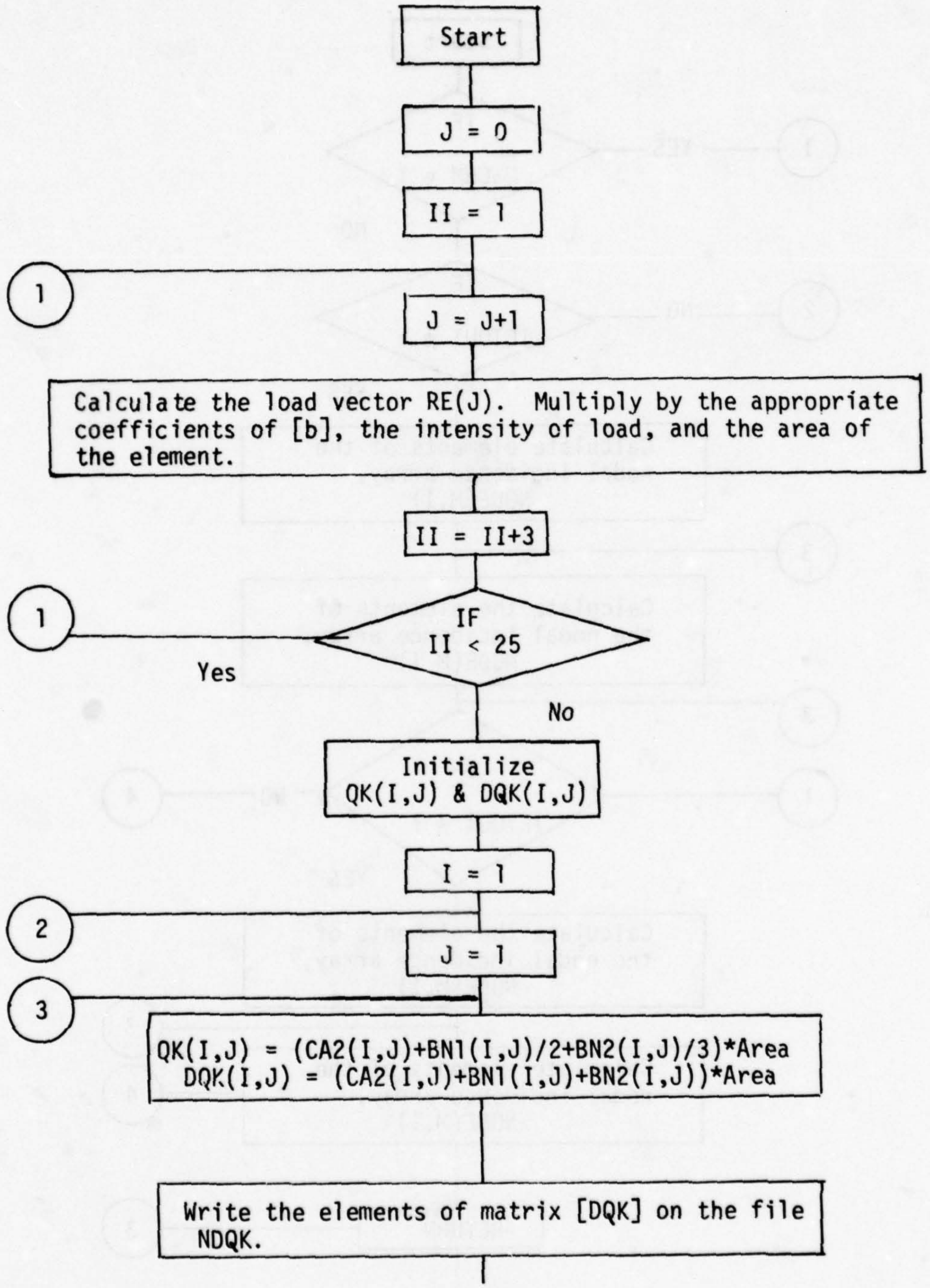
Set the diagonal element corresponding to ID(k) equal to 1. Also set the row and column elements corresponding to ID(k) equal to zero. These elements are from the block under consideration only.
6
1
K = K+1
3
No
IF k>IK
Yes
IF KEX = 1
No
Yes
Write the elements of the previously completely processed block on tape ND.
Write the elements of the previously completely processed block and MAXA on working file ND.
N = N+1
4
No
IF N>NBLOCK
Yes

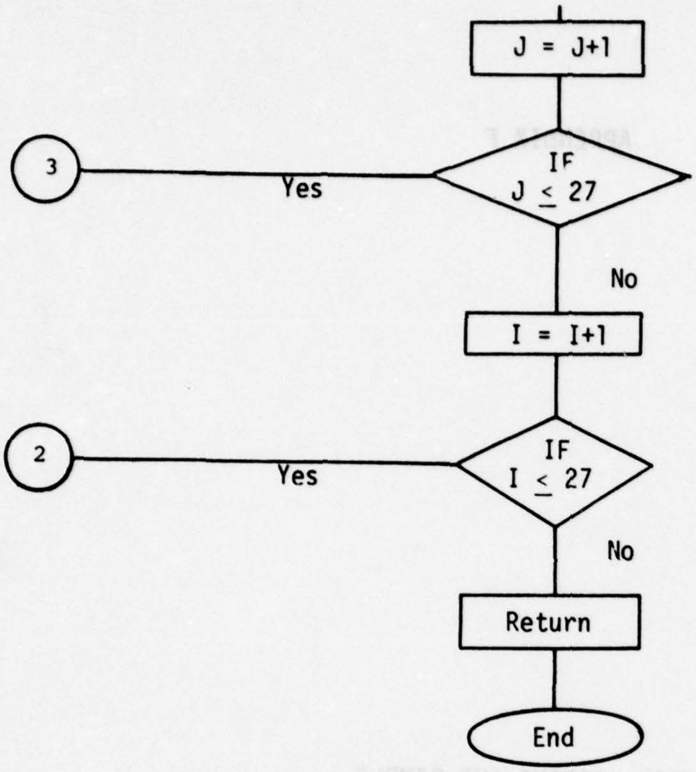

IF
KEX = 1
No
Yes
Write the elements of the last block on the work file ND
Write the elements of the last block and MAXA on the work file ND
RETURN
END

SUBROUTINE INCDNC



SUBROUTINE ELMNT




J = J+1
3
Yes
IF
J ≤ 27
No
I = I+1
2
Yes
IF
I ≤ 27
No
Return
End

APPENDIX F

PROGRAM LISTING AND SAMPLE PROBLEM INPUT DATA

```
C
C
C     ****************************************************************
C                              C L A P P
C            COLLAPSE LOAD ANALYSIS FOR PANELS AND PLATES
C     THIS PROGRAM CALCULATES THE COLLAPSE LOAD FOR PLATES AND PANELS.
C     WRITTEN IN FORTRAN G1 LEVEL FOR THE IBM 370/165 MODEL COMPUTER
C     AT CLEMSON UNIVERSITY AT CLEMSON, SOUTH CAROLINA. WRITTEN BY
C     DR. NELSON R. BAULD  AND MR. KAILASAM SATYAMURTHY.
C
C
C     ****************************************************************
C     MAXIMUM NUMBER OF ELEMENTS = 244
C     MAXIMUM NUMBER OF NODAL POINTS = 240
C     MAXIMUM BAND WIDTH PERMITTED = 81
C     ****************************************************************
C
      DIMENSION QK(27,27),DQK(27,27),ELN(6,6),CQ(12),B(6,9),RE(27)
      DIMENSION CA2(27,27),NBC(9),PA2(12,12),AN1(12,12),PA3(12,12,12)
      DIMENSION BN1(27,27),BN2(27,27),LP(27)
      DIMENSION P33(9,9),P34(9,9),P35(9,9),P38(9,9),P39(9,9)
      DIMENSION P44(9,9),P45(9,9),P48(9,9),P49(9,9),P55(9,9)
      DIMENSION P58(9,9),P59(9,9),P88(9,9),P89(9,9),P99(9,9)
      DIMENSION PA2T(12,12),PA3T(12,12,12)
C
C
C
      DIMENSION ID(400),CC(100),X(240),Y(240),NODE(244,9),ITYPE(244)
      DIMENSION PLOAD(244,3),PINCR(244,3),PINCO(244,3),IBC(244,3)
      DIMENSION MAXA(170),MAXB(170),AK(246,82),P1(6724),NODES(244)
      DIMENSION P2(6724),D(6724),SC(10),Q(850),P6(100)
      DIMENSION HED(18)
      EQUIVALENCE (AK(1),P1(1)),(AK(6725),P2(1)),(AK(13449),D(1))
      COMMON /P11/ QK,DQK,RE,B,CA2
      COMMON /P12/ BN1,BN2,PLOAD,PINCR
      COMMON /P13/ PA2,PA3
      COMMON /P14/ ELN,AREA,R
      COMMON /P15/ X,Y,NODE,ITYPE,IBC,NODES
      COMMON /P16/ PA2T,PA3T
      COMMON /P17/ ID,IK,NBC
      COMMON /P19/ P33,P34,P35,P38,P39,P44,P45,P48,P49,P55
      COMMON /P20/ P58,P59,P88,P89,P99
C
C
C
      READ (1,422) IRUN
  422 FORMAT (I5)
      WRITE (3,423) IRUN
  423 FORMAT (3X,'RUN NUMBER = ',2X,I5)
      NTAPE = 25
C
      KEX=1
      FM=1.0
      ICOUNT=0
```

```
      KCUT=0
C
      MNR=1
      NTIME=3
      NINE=9
      INIT=0
      ITER=1
      LEVEL=1
      ITMAX=5
      LEMAX=25
      NRED=15
      NR=16
      NTHREE=3
      NEOB=82
      NSTIF=10
      KSTIF=11
      NX1=12
      NL=13
      NC=14
      NDIS=17
      ND=18
      NC2=20
      IPA3T=21
      NBN1=22
      NBN2=23
      NDOK=24
      REWIND NDOK
      REWIND NBN1
      REWIND NBN2
      REWIND IPA3T
      REWIND  NC2
      REWIND ND
      REWIND NDIS
      REWIND NX1
      REWIND NSTIF
      REWIND KSTIF
      REWIND NL
      REWIND NC
      REWIND NTAPE
C     ****************************************************************
C     THIS READS THE TOTAL NUMBER OF ELEMENTS AND THE TOTAL NUMBER
C     OF NODAL POINTS. NEL=TOTAL NUMBER OF ELEMENTS.
C     NUMNP=TOTAL NUMBER OF NODAL POINTS.
C     IEX IS AN INDEX CORRESPONDS TO THE INITIAL IMPERFECTION.
C     IEX = 1 INITIAL IMPERFECTION IS PRESENT.
C     IEX = 0 NO INITIAL IMPERFECTION.
C     ****************************************************************
C
      WRITE (3,1300)
 1300 FORMAT (/// 25X,
     1'***********************************************************'//)
      READ (1,20) HED,NEL,NUMNP,IEX,IBIF,NFLAT
   20 FORMAT (18A4/2I10,3I5)
      WRITE (3,39) HED,NEL,NUMNP,IEX,IBIF,NFLAT
```

```
   39 FORMAT (1H1,25X,18A4///
     1 25X,30H* NUMBER OF ELEMENTS---------- I10 //
     2 25X,30H* NUMBER OF NODAL POINTS------ I10 //
     3 25X,30H* INITIAL IMPERFECTION INDEX-- I5 //
     4 25X,30H* BIFURCATION INDEX----------- I5 //
     5 25X,30H* FLAT PLATE ANALYSIS INDEX-- I5 //)
C     ****************************************************************
C     THIS READS THE RADIUS OF CURVATURE AND THE ACCURACY CRITERION
C     EPSILON,WHICH IS USED IN THE NEWTON-RAPHSON ITERATION SCHEME.
C     *****************************************************************
      READ (1,21) R,EPSI
   21 FORMAT (F10.0,F14.0)
      WRITE (3,22) R,EPSI
   22 FORMAT(1H0,
     1 25X,30H*RADIUS OF CURVATURE---------- F10.4 ,2X,'INCHES' //
     1 25X,36H* CONVERGENCE CRITERION EPSILON----- F10.6//)
C     ****************************************************************
C     THE INITIAL IMPERFECTION IS OF THE FORM W = W0*SIN (M*PI*X/A) *
C     SIN (N*PI*Y/B), WHERE W0 IS THE MAXIMUM INITIAL IMPERFECTION AT
C     THE CENTER, CONS1= (M*PI/A), CONS2= (N*PI/B). X0 AND Y0 ARE THE
C     OFFSETS FROM THE X AND Y AXES RESPECTIVELY.
C     ****************************************************************
      READ (1,11) W0,CONS1,CONS2,X0,Y0
   11 FORMAT (5F10.0)
      WRITE (3,12) W0,CONS1,CONS2,X0,Y0
   12 FORMAT (1H0,
     1 25X,30H*MAX.IMPERFECTION AMPLITUDE---   F16.6,2X,'INCHES' //
     2 25X,30H* WAVE NUMBER IN X-DIRECTION-  F16.6 //
     3 25X,30H* WAVE NUMBER IN Y-DIRECTION-   F16.6 //
     4 25X,30H* OFFSET FROM THE X-AXIS-----   F16.6,2X,'INCHES' //
     5 25X,30H* OFFSET FROM THE Y-AXIS-----   F16.6,2X,'INCHES'//)
      WRITE (3,1300)
C     *****************************************************************
C     THIS READS THE ELEMENT NUMBER,THE TYPE OF THE ELEMENT AND
C     THE NODAL INCIDENCES ASSOCIATED WITH EACH ELEMENT. EVERY
C     ELEMENT HAS NINE NODAL INCIDENCES ASSOCIATED WITH IT.
C     ******************************************************************
C
      WRITE (3,1200)
 1200 FORMAT (1H1,2X,'EL.NO.',2X,'TYPE',2X,'NODE-1',2X,'NODE-2',2X,'NODE
     1-3',2X,'NODE-4',2X,'NODE-5',2X,'NODE-6',2X,'NODE-7',2X,'NODE-8',
     22X,'NODE-9')
C*************************************************************
C     HERE THE QUANTITIES NECESSARY FOR THE MESH GENERATION ARE READ.
C     NSCHM IS EQUAL TO 1 IF THE FINITE-DIFFERENCE NODES ARE NUMBER-
C     ED CONSECUTIVELY IN THE Y- DIRECTION.
C     IT IS NOT EQUAL TO 1 IF THE FINITE-DIFFERENCE NODES ARE NUMBER-
C     ED CONSECUTIVELY IN THE X- DIRECTION.
C     ICTOUT IS EQUAL TO 1 FOR A PANEL WITH A CUT-OUT.
C     ICTOUT IS NOT EQUAL TO 1 FOR A PANEL WITHOUT A CUT-OUT.
C     NROWS - NUMBER OF ROWS OF FINITE-DIFFERENCE GRID POINTS - A ROW
C     BEING A LINE OF CONSECUTIVELY NUMBERED NODES.
C     NCOLS - NUMBER OF COLUMNS OF FINITE-DIFFERENCE GRID POINTS.
C     IROW1, IROW2 - NUMBER OF THE ROWS IN WHICH THE SIDES OF THE
```

```
C      CUT-OUT PARALLEL TO THE ROWS OF FINITE-DIFFERENCE GRID
C      POINTS LIE.
C      ICOL1, ICOL2 - NUMBERS OF THE COLUMNS IN WHICH THE SIDES OF
C      OF THE CUT-OUT PARALLEL TO THE COLUMNS OF THE FINITE-DIFFERE-
C      NCE GRID POINTS LIE.
C      NOTE: NODE NUMBERING SHOULD ALWAYS PROCEED FROM THE UPPER
C      LEFT-HAND CORNER OF THE FINITE-DIFFERENCE GRID.
C      ******************************************************************
       READ (1,641) NSCHM,ICTOUT,NCOLS,NROWS,IROW1,IROW2,ICOL1,ICOL2
  641  FORMAT (8I5)
       READ (1,642) (ITYPE(I),I=1,NEL)
  642  FORMAT (10(I5))
       READ (1,643) (NODES(I),I=1,NEL)
  643  FORMAT (10(I5))
       CALL INCDNC (NEL,NROWS,NCOLS,IROW1,IROW2,ICOL1,ICOL2,NSCHM,ICTOUT)
       WRITE (3,25) ((M,ITYPE(M),(NODE(M,I),I=1,9)),M=1,NEL)
   25  FORMAT (3I7,8I8)
C      HERE THE SUBROUTINE IBAND IS CALLED TO DETERMINE THE BAND
C      WIDTH OF THE SYSTEM.
       CALL IBAND(NEL,LBAND)
       WRITE (3,1300)
       WRITE (3,26) LBAND
   26  FORMAT (1H0,25X, 'BAND WIDTH OF THE SYSTEM =',I5)
       WRITE (3,1300)
       WRITE (3,1201)
 1201  FORMAT (1H1,5X,'NODAL POINT',4X,'X-COORDINATE',6X,'Y-COORDINATE')
C      HERE THE X AND Y COORDINATES OF EACH NODAL POINT IS READ.
       READ (1,27) ((N,X(N),Y(N)),N=1,NUMNP)
   27  FORMAT (I5,2F10.0)
       WRITE (3,28) ((N,X(N),Y(N)),N=1,NUMNP)
   28  FORMAT (I12,3X,F16.6,2X,F16.6)
       WRITE (3,1202)
 1202  FORMAT (1H1,6X,'ELEMENT NO.',5X,'Z-LOAD',10X,'X-LOAD',10X,
      1'Y-LOAD')
       READ (1,15) ((M,(PLOAD(M,I),I=1,3)),M=1,NEL)
   15  FORMAT (I5,3F10.0)
       WRITE (3,16) ((M,(PLOAD(M,I),I=1,3)),M=1,NEL)
   16  FORMAT (I15,2X,3F16.6)
       READ (1,17) ZINCR,XINCR,YINCR
   17  FORMAT (3F10.0)
       WRITE (3,18) ZINCR,XINCR,YINCR
   18  FORMAT (1H1,
      1 2X,'LOAD INCREMENT IN Z-DIRECTION--',F16.6,2X,'POUNDS' /
      2 2X,'LOAD INCREMENT IN X-DIRECTION--',F16.6,2X,'POUNDS' /
      3 2X,'LOAD INCREMENT IN Y-DIRECTION--',F16.6,2X,'POUNDS')
       WRITE (3,1203)
 1203  FORMAT (1H0,5X,'ELEMENT NO.',5X,'Z-BOUNDARY',5X,'X-BOUNDARY',5X,
      1'Y-BOUNDARY')
       READ (1,33)      ((M,(IBC(M,I),I=1,3)),M=1,NEL)
       WRITE (3,34)     ((M,(IBC(M,I),I=1,3)),M=1,NEL)
   33  FORMAT (I5,3I5)
   34  FORMAT (I12,I16,I15,I15)
C      ******************************************************************
C      HERE THE SUBROUTINE ELPROP IS CALLED TO DETERMINE THE MATERIAL
```

```
C     STIFFNESS MATRIX OF THE SYSTEM.
C     ******************************************************************
      CALL ELPROP
      WRITE (3,1204)
 1204 FORMAT (1H0,10X,'THE COMPOSITE MATERIAL STIFFNESS MATRIX' /)
      WRITE (3,511) ((EIN(I,J),J=1,6),I=1,6)
  511 FORMAT (5X,6F16.6)
      NEQ= 3*NUMNP
      ANBLK=FLOAT(NEQ)/FLOAT(NEQB)
      IF (ANBLK .GT. INT(ANBLK)) GO TO 9922
      NBLOCK=INT(ANBLK)
      GO TO 9923
 9922 NBLOCK=INT(ANBLK)+1
 9923 WRITE (3,9924) NBLOCK
 9924 FORMAT (1H0,10X,'TOTAL NUMBER OF BLOCKS =',I5)
      DO 7059 M=1,NEL
      DO 7059 I=1,3
      PINCR(M,I)=0.0
      IF (PLOAD(M,I) .EQ. 0.0 ) PINCO(M,I)=0.0
      IF (PLOAD(M,1) .NE. 0.0 ) PINCO(M,1)=ZINCR
      IF (PLOAD(M,2) .NE. 0.0 ) PINCO(M,2)=XINCR
      IF (PLOAD(M,3) .NE. 0.0 ) PINCO(M,3)=YINCR
 7059 CONTINUE
      CALL COEFF (NFLAT)
      LB1=LBAND+1
      NAV=NEQB*LB1
      MI=LBAND+NEQB-1
 1532 DO 530 I=1,850
  530 Q(I)=0.0
      CALL FDIFF (NEL,NUMNP,NX1,NC2,W0,CONS1,CONS2,X0,Y0,IEX,IPA3T,NFLAT)
C     ******************************************************************
      IF (IRUN .EQ. 1) GO TO 7054
  421 REWIND NTAPE
      READ (NTAPE) ((PLOAD(M,I),I=1,3),M=1,NEL)
      READ (NTAPE) ((PINCO(M,I),I=1,3),M=1,NEL)
      READ (NTAPE) ((PINCR(M,I),I=1,3),M=1,NEL)
      READ (NTAPE) (Q(I),I=1,NEQ)
      READ (NTAPE) LEVEL,ICOUNT,KCUT
      WRITE (3,470)
  470 FORMAT (2X,'THE LOADS ARE')
      WRITE (3,1202)
      WRITE (3,471) ((M,(PLOAD(M,I),I=1,3)),M=1,NEL)
  471 FORMAT (I15,2X,3F16.6)
 7054 REWIND NX1
      REWIND NSTIF
      REWIND KSTIF
      REWIND NDIS
      REWIND NC2
      REWIND IPA3T
      REWIND NBN1
      REWIND NBN2
      REWIND NDQK
      ICOUNT=ICOUNT+1
      NNB=2*NEQB
```

```
      NUM3P=NUMNP*3
      ND2=2*NEQB
      ND3=3*NEQB
      LBLOCK=0
      NTIMES=MNR*NTIME
      M=1
      DO 1002 I=1,ND3
      DO 1002 J=1,NEQB
 1002 AK(I,J)=0.0
      MML=0
  710 NNB=NNB+NEQB
      LBLOCK=LBLOCK+1
      KB=LBLOCK
      ML=M
      IF (MML .GT. NEL) GO TO 707
      DO 200 M=ML,NEL
      IF ( 3*NODE(M,9) .GT. NNB) GO TO 707
      READ (NX1) B,AREA
      MML=M+1
      DO 73 I=3,27,3
      IJ=I/3
      LP(I-2)=3*NODE(M,IJ)-2
      LP(I-1)=3*NODE(M,IJ)-1
   73 LP(I)=3*NODE(M,IJ)
      K1=LP(1)
      I1=LP(2)
      J1=LP(3)
      K2=LP(4)
      I2=LP(5)
      J2=LP(6)
      K3=LP(7)
      I3=LP(8)
      J3=LP(9)
      K4=LP(10)
      I4=LP(11)
      J4=LP(12)
      K5=LP(13)
      I5=LP(14)
      J5=LP(15)
      K6=LP(16)
      I6=LP(17)
      J6=LP(18)
      K7=LP(19)
      I7=LP(20)
      J7=LP(21)
      K8=LP(22)
      I8=LP(23)
      J8=LP(24)
      K9=LP(25)
      I9=LP(26)
      J9=LP(27)
      READ (NC2) CA2
      DO 900 I=1,3
  900 CQ(I)=B(I,1)*Q(I1)+B(I,2)*Q(I2)+B(I,3)*Q(I3)+B(I,4)*Q(I4)+
```

```
     1B(I,5)*Q(I5)+B(I,6)*Q(I6)+B(I,7)*Q(I7)+B(I,8)*Q(I8)+B(I,9)*Q(I9)
      DO 901 I=1,3
  901 CQ(I+3)=B(I,1)*Q(J1)+B(I,2)*Q(J2)+B(I,3)*Q(J3)+B(I,4)*Q(J4)+
     1B(I,5)*Q(J5)+B(I,6)*Q(J6)+B(I,7)*Q(J7)+B(I,8)*Q(J8)+B(I,9)*Q(J9)
      DO 902 I=1,6
  902 CQ(I+6)=B(I,1)*Q(K1)+B(I,2)*Q(K2)+B(I,3)*Q(K3)+B(I,4)*Q(K4)+
     1B(I,5)*Q(K5)+B(I,6)*Q(K6)+B(I,7)*Q(K7)+B(I,8)*Q(K8)+B(I,9)*Q(K9)
      IF (IEX .EQ. 1) GO TO 6
      DO 904 I=1,12
      DO 904 J=1,12
      CSUM=PA3(I,J,1)*CQ(1)+PA3(I,J,2)*CQ(2)+PA3(I,J,3)*CQ(3)
      ESUM=PA3(I,J,4)*CQ(4)+PA3(I,J,5)*CQ(5)+PA3(I,J,6)*CQ(6)
      FSUM=PA3(I,J,7)*CQ(7)+PA3(I,J,8)*CQ(8)+PA3(I,J,9)*CQ(9)
      GSUM=PA3(I,J,10)*CQ(10)+PA3(I,J,11)*CQ(11)+PA3(I,J,12)*CQ(12)
  904 AN1(I,J)=3.0*(CSUM+ESUM+FSUM+GSUM)
      CALL ATBA (AN1,B,BN1)
      GO TO 7
    6 READ (IPA3T) PA3T
      DO 906 I=1,12
      DO 906 J=1,12
      CSUM=PA3T(I,J,1)*CQ(1)+PA3T(I,J,2)*CQ(2)+PA3T(I,J,3)*CQ(3)
      ESUM=PA3T(I,J,4)*CQ(4)+PA3T(I,J,5)*CQ(5)+PA3T(I,J,6)*CQ(6)
      FSUM=PA3T(I,J,7)*CQ(7)+PA3T(I,J,8)*CQ(8)+PA3T(I,J,9)*CQ(9)
      GSUM=PA3T(I,J,10)*CQ(10)+PA3T(I,J,11)*CQ(11)+PA3T(I,J,12)*CQ(12)
  906 AN1(I,J)=3.0*(CSUM+ESUM+FSUM+GSUM)
      CALL ATBA (AN1,B,BN1)
    7 DO 920 I=1,12
      DO 920 J=1,12
  920 AN1(I,J)=0.0
      DO 905 I=1,9
      DO 905 J=1,9
      CSUM=CQ(3)**2*P33(I,J)+CQ(3)*CQ(4)*2.0*P34(I,J)
      ESUM=CQ(3)*2.0*CQ(5)*P35(I,J)+CQ(3)*2.0*CQ(8)*P38(I,J)
      FSUM=CQ(3)*2.*CQ(9)*P39(I,J)+CQ(4)**2*P44(I,J)
      GSUM=2.0*CQ(4)*(CQ(5)*P45(I,J)+P48(I,J)*CQ(8))
      HSUM=2.0*CQ(4)*CQ(9)*P49(I,J)+CQ(5)**2*P55(I,J)
      PSUM=2.0*CQ(5)*(P58(I,J)*CQ(8)+P59(I,J)*CQ(9))
      QSUM=P88(I,J)*CQ(8)**2+2.0*CQ(8)*CQ(9)*P89(I,J)
      QSUM=QSUM+P99(I,J)*CQ(9)**2
  905 AN1(I,J)=6.0*(CSUM+ESUM+FSUM+GSUM+HSUM+PSUM+QSUM)
      DO 921 I=1,8
      JL=I+1
      DO 921 J=JL,9
  921 AN1(J,I)=AN1(I,J)
      CALL ATBA (AN1,B,BN2)
      IF (LEVEL .EQ. 2 .AND. ITER .EQ. 1) GO TO 711
      GO TO 701
  711 IF (IBIF .NE. 1) GO TO 701
      WRITE (NBN1) BN1
      WRITE (NBN2) BN2
C
C
C
  701 CALL ELMNT(NEL,M,NDQK)
```

```
C     THIS CODING ASSEMBLES THE COEFFICIENTS OF THE BOUNDARY ELEMENTS
C     IN THE MASTER STIFFNESS MATRIX AND STORES THEM IN AN ARRAY OF
C     DIMENSIONS NEQ X IBAND.
   60 DO 40 I=3,27,3
      IJ=I/3
      LP(I-2)=3*NODE(M,IJ)-2
      LP(I-1)=3*NODE(M,IJ)-1
   40 LP(I)=3*NODE(M,IJ)
      DO 50 LL=1,27
      I=LP(LL)
      II=I-(KB-1)*NEQB
      AK(II,LB1)=AK(II,LB1)+RE(LL)
      DO 50 MM=1,27
      J=LP(MM)-I+1
      IF(J .LE. 0)GO TO 50
      AK(II,J)=AK(II,J)+QK(LL,MM)
   50 CONTINUE
  200 CONTINUE
C
C
  707 WRITE (NSTIF) ((AK(I,J),I=1,NEQB),J=1,LB1)
      DO 705 NN=1,ND2
      DO 705 MM=1,LB1
      KK=NN+NEQB
  705 AK(NN,MM)=AK(KK,MM)
      ND2P1=ND2+1
      DO 706 NN=ND2P1,ND3
      DO 706 MM=1,LB1
  706 AK(NN,MM)=0.0
      IF(NBLOCK - LBLOCK)  1600,1600,710
 1600 REWIND NSTIF
      REWIND NDQK
      MNR=1
      NNB=2*NEQB
      LBLOCK=0
      NTIMES=MNR*NTIME
      M=1
      DO 1601 I=1,ND3
      DO 1601 J=1,NEQB
 1601 AK(I,J)=0.0
      MML=0
 1602 NNB=NNB+NEQB
      LBLOCK=LBLOCK+1
      KB=LBLOCK
      ML=M
      IF (MML .GT. NEL) GO TO 1603
      DO 1604 M=ML,NEL
      IF ( 3*NODE(M,9) .GT. NNB) GO TO 1603
      MML=M+1
      READ (NDQK) DQK
 1605 DO 1606 I=3,27,3
      IJ=I/3
      LP(I-2)=3*NODE(M,IJ)-2
      LP(I-1)=3*NODE(M,IJ)-1
```

```
1606 LP(I)=3*NODE(M,IJ)
     DO 1607 LL=1,27
     I=LP(LL)
     II=I-(KB-1)*NEQB
     DO 1607 MM=1,27
     J=LP(MM)-I+1
     IF (J .LE. 0) GO TO 1607
     AK(II,J)=AK(II,J)+DQK(LL,MM)
1607 CONTINUE
1604 CONTINUE
1603 WRITE (KSTIF) ((AK(I,J),I=1,NEQB),J=1,LB1)
     DO 1608 NN=1,ND2
     DO 1608 MM=1,LB1
     KK=NN+NEQB
1608 AK(NN,MM)=AK(KK,MM)
     ND2P1=ND2+1
     DO 1609 NN=ND2P1,ND3
     DO 1609 MM=1,LB1
1609 AK(NN,MM)=0.0
     IF (NBLOCK - LBLOCK) 1610,1610,1602
1610 REWIND KSTIF
     NEQ=3*NUMNP
     CALL BCID(NEL,NEQ,NBLOCK,NEQB,KBLOCK,NN)
     WRITE (3,1112) KBLOCK,NN
1112 FORMAT (1H0,2X,'KBLOCK=',I5,'NN=',I5)
1111 IF (INIT .GE. 1) GO TO 7030
     DO 7031 K=1,NBLOCK
     IL=(K-1)*NEQB+1
     IR=K*NEQB
7031 WRITE (NDIS) (Q(I),I=IL,IR)
     INIT=1
7030 NAR=NEQB*NBLOCK+LBAND
     IF ( KEX .NE. 1) GO TO 7090
     REWIND NC
     REWIND NSTIF
     REWIND KSTIF
     CALL MULTOC(P1,P2,P6,D,Q,MAXA,NAV,NEQB,NAR,LBAND,NBLOCK,MI,NSTIF,
    1KSTIF,NC,1)
     REWIND NC
     REWIND KSTIF
     CALL BCS(P1,P2,MAXA,NAV,NEQB,LBAND,MI,1,NBLOCK,NC,KSTIF,MAXB)
     REWIND KSTIF
     REWIND NRED
     REWIND NR
     REWIND NL
     NEQ=3*NUMNP
     CALL SESOL(P1,P2,MAXA,NEQ,LBAND,1,NBLOCK,NEQB,NAV,MI,KSTIF,NRED,
    1NL,NR,KBLOCK,NN,1,MAXB,DET)
     GO TO 7091
7090 REWIND NSTIF
     REWIND NRED
     REWIND NC
     CALL MULTOC(P1,P2,P6,D,Q,MAXA,NAV,NEQB,NAR,LBAND,NBLOCK,MI,
    1NSTIF,NRED,NC,2)
```

```
      REWIND NC
      REWIND NRED
      CALL BCS(P1,P2,MAXA,NAV,NEQB,LBAND,MI,2,NBLOCK,NC,NRED,MAXB)
      REWIND KSTIF
      REWIND NRED
      REWIND NL
      REWIND NR
      CALL SESOL(P1,P2,MAXA,NEQ,LBAND,1,NBLOCK,NEQB,NAV,MI,KSTIF,NRED,
     1NL,NR,KBLOCK,NN,2,MAXB,DET)
 7091 REWIND NL
      REWIND NDIS
      CALL SMALL(Q,CC,SC,NEQB,NBLOCK,LEVEL,ITER,BIG,NL,NDIS,NAR,NUMNP)
      WNORM=0.
      NUM3P=NUMNP*3
      DO 800 I=1,NUM3P,3
  800 WNORM=WNORM+Q(I)**2
      WNORM=SQRT(WNORM)
      WRITE(3,801) WNORM
  801 FORMAT(1H0,2X,'THE NORMALIZED  TRANSVERSE DISPLACEMENT IS =',
     1F14.8)
C     ******************************************************************
      IF (LEVEL .EQ. 2 .AND. ITER .EQ. 1) GO TO 1122
      GO TO 1123
 1122 IF (IBIF .NE. 1) GO TO 1123
      DO 810 M=1,NEL
      DO 810 I=1,3
  810 PINCR(M,I)=0.5*PLOAD(M,I)
  827 REWIND NC2
      REWIND NX1
      REWIND NSTIF
      REWIND KSTIF
      REWIND NBN1
      REWIND NBN2
      NNB=2*NEQB
      NUM3P=NUMNP*3
      ND2=2*NEQB
      ND3=3*NEQB
      LBLOCK=0
      M=1
      DO 811 I=1,ND3
      DO 811 J=1,NEQB
  811 AK(I,J)=0.0
      MML=0
  820 NNB=NNB+NEQB
      LBLOCK=LBLOCK+1
      KB=LBLOCK
      ML=M
      IF (MML .GT. NEL) GO TO 812
      DO 813 M=ML,NEL
      IF (3*NODE(M,9) .GT. NNB) GO TO 812
      READ (NX1) B,AREA
      MML=M+1
      READ (NBN1) BN1
      READ (NBN2) BN2
```

```
      READ (NC2) CA2
      DO 814 I=1,27
      DO 814 J=1,27
  814 QK(I,J)=(CA2(I,J)+FM*BN1(I,J)+FM*FM*BN2(I,J))*AREA
  821 DO 815 I=3,27,3
      IJ=I/3
      LP(I-2)=3*NODE(M,IJ)-2
      LP(I-1)=3*NODE(M,IJ)-1
  815 LP(I)=3*NODE(M,IJ)
      DO 816 LL=1,27
      I=LP(LL)
      II=I-(KB-1)*NEQB
      DO 816 MM=1,27
      J=LP(MM)-I+1
      IF (J .LE. 0) GO TO 816
      AK(II,J)=AK(II,J)+QK(LL,MM)
  816 CONTINUE
  813 CONTINUE
  812 WRITE (NSTIF) ((AK(I,J),I=1,NEQB),J=1,LB1)
      DO 817 NN=1,ND2
      DO 817 MM=1,LB1
      KK=NN+NEQB
  817 AK(NN,MM)=AK(KK,MM)
      ND2P1=ND2+1
      DO 818 NN=ND2P1,ND3
      DO 818 MM=1,LB1
  818 AK(NN,MM)=0.0
      IF (NBLOCK - LBLOCK) 819,819,820
  819 REWIND NSTIF
      NEQ=NUMNP*3
      CALL BCID(NEL,NEQ,NBLOCK,NEQB,KBLOCK,NN)
      WRITE (3,822) KBLOCK,NN
  822 FORMAT (2X,'KBLOCK=',I5,'NN=',I5)
      CALL BCS(P1,P2,MAXA,NAV,NEQB,LBAND,MI,1,NBLOCK,NSTIF,KSTIF,MAXB)
      REWIND KSTIF
      REWIND NRED
      REWIND NL
      REWIND NR
      CALL SESOL(P1,P2,MAXA,NEQ,LBAND,1,NBLOCK,NEQB,NAV,MI,KSTIF,
     1NRED,NL,NR,KBLOCK,NN,1,MAXB,DET)
      FM=FM+0.5
      IF (DET .LE. 0.0) GO TO 823
      DO 824 M=1,NEL
      DO 824 I=1,3
  824 PLOAD(M,I)=PLOAD(M,I)+PINCR(M,I)
      WRITE (3,825)
  825 FORMAT (1H1,6X,'ELEMENT NO.',5X,'Z-LOAD',10X,'X-LOAD',10X,
     1'Y-LOAD' //)
      WRITE (3,826) ((PLOAD(M,I),I=1,3),M=1,NEL)
  826 FORMAT (15X,2X,3F16.6)
      GO TO 827
  823 WRITE (3,828)
  828 FORMAT (1H0,4X,'**********************************************')
      WRITE (3,829)
```

```
  829 FORMAT (1H0,10X,'THE SYSTEM HAS BUCKLED DURING THE PREVIOUS LOAD
     1LEVEL')
      WRITE (3,830)
  830 FORMAT (4X,'THE TOTAL BUCKLING LOAD IS OBTAINED AS FOLLOWS:' /)
      WRITE (3,831)
  831 FORMAT (4X,'THE APPLIED PRESSURE DURING THE LAST LOAD LEVEL IS ')
      WRITE (3,832)
  832 FORMAT (4X,'MULTIPLIED BY THE CORRESPONDING ELEMENT AREA' /)
      WRITE (3,833)
  833 FORMAT (4X,'THE TOTAL BUCKLING LOAD IS THE SUM OF ALL THE
     1INDIVIDUAL ELEMENT LOADS'/)
      WRITE (3,828)
      GO TO 520
 1123 IF(ABS(BIG) .LE. EPSI) GO TO 7052
      IF(ITER .GE. ITMAX) GO TO 7053
      KEX=2
      IF (ITER .EQ. NTIMES) KEX=1
      ITER=ITER+1
      IF (KEX .NE. 1) GO TO 7054
      MNR=MNR+1
      GO TO 7054
 7052 LEVEL=LEVEL+1
      IF (IBIF .EQ. 1) GO TO 911
 9995 IF(LEVEL .GT. LEMAX) GO TO 7053
      IF (KCUT .GE. 1) GO TO 7070
      IF (ICOUNT-1) 7060,7060,7061
 7060 ZN=1.0
      GO TO 7062
 7061 ZN=0.5
      IF (LEVEL .EQ. 2 .AND. ITER .GE. 2) GO TO 7062
      IF (ITER .GE. 3) GO TO 7063
 7062 DO 7058 M=1,NEL
      DO 7058 I=1,3
      PINCR(M,I)=     PINCR(M,I)+ZN*PINCO(M,I)
 7058 PLOAD(M,I)=PLOAD(M,I)+PINCR(M,I)
      GO TO 7065
 7063 ZN=1.
      IF (ITER .GE. 4) GO TO 7070
 7071 DO 7064 M=1,NEL
      DO 7064 I=1,3
      PINCR(M,I)=ZN*PINCO(M,I)
 7064 PLOAD(M,I)=PLOAD(M,I)+PINCR(M,I)
 7065 CONTINUE
      GO TO 7075
 7070 KCUT=KCUT+1
      ZN=0.5
      IF (KCUT .GT. 1 .AND. ITER .LE. 3) GO TO 7071
      ZN=ZN/2.0
      GO TO 7071
 7075 WRITE (3,7042)
 7042 FORMAT(2X,'THE LOADS ARE')
      WRITE (3,1202)
      WRITE (3,32) ((M,(PLOAD(M,I),I=1,3)),M=1,NEL)
   32 FORMAT (I15,2X,3F16.6)
```

```
      ITER=1
      MNP=1
      KEX=1
      REWIND NTAPE
      WRITE (NTAPE)  ((PLOAD(M,I),I=1,3),M=1,NEL)
      WRITE (NTAPE)  ((PINCO(M,I),I=1,3),M=1,NEL)
      WRITE(NTAPE)  ((PINCR(M,I),I=1,3),M=1,NEL)
      WRITE (NTAPE)  (Q(I),I=1,NEQ)
      WRITE (NTAPE) LEVEL,ICCUNT,KCUT
  911 GO TO 7054
 7053 WRITE(3,7055)
 7055 FORMAT(1H0,2X,'NUMBER OF ITERATIONS EXCEEDED ITMAX')
  520 CONTINUE
C
      STOP
      END
      SUBROUTINE ELMNT(NEL,M,NDQK)
C     THIS SUBROUTINE ELMNT CALCULATES THE STIFFNESS MATRIX OF
C     AND ITS DERIVATIVE FOR EVERY ELEMENT.THEY ARE QK (27,27)
C     AND DQK(27,27) RESPECTIVELY. THIS ALSO CALCULATES THE LOAD
C     VECTOR RE(27) FOR EVERY ELEMENT BY MULTIPLYING THE LOAD
C     INTENSITY BY THE AREA OF THAT PARTICULAR ELEMENT.
C
      DIMENSION QK(27,27),DQK(27,27),RE(27),B(6,9),CA2(27,27)
      DIMENSION BN1(27,27),BN2(27,27),PLOAD(244,3),ELN(6,6),PINCR(244,3)
      COMMON /P11/ QK,DQK,RE,B,CA2
      COMMON /P12/ BN1,BN2,PLOAD,PINCR
      COMMON /P14/ ELN,AREA,R
      J=0
      DO 71 II=1,25,3
      J=J+1
      RE(II)=B(1,J)*PLOAD(M,1)*AREA
      RE(II+1)=B(1,J)*PLOAD(M,2)*AREA
   71 RE(II+2)=B(1,J)*PLOAD(M,3)*AREA
      DO 70 I=1,27
      DO 70 J=1,27
      QK(I,J)=0.0
   70 DQK(I,J)=0.0
      DO 75 I=1,27
      DO 75 J=1,27
      QK(I,J)=(CA2(I,J)+BN1(I,J)/2.0+BN2(I,J)/3.0)*AREA
   75 DQK(I,J)=(CA2(I,J)+BN1(I,J)+BN2(I,J))*AREA
      WRITE (NDQK) DQK
   76 RETURN
      END
C
      SUBROUTINE COEFF (NFLAT)
C      THIS SUBROUTINE COEFF CALCULATS THE COEFFICIENTS PA2(12,12)
C      WHICH ARE ASSOCIATED WITH QUADRATIC IN DISPLACEMENTS,
C      PA3(12,12,12) ASSOCIATED WITH CUBIC DISPLACEMENTS AND
C      PA4(12,12,12,12) WHICH ARE ASSOCIATED WITH QUARTIC DISPLACE-
C      MENTS. THESE COEFFICIENTS DEPEND ONLY ON THE MATERIAL PROPERTY
C      AND THE RADIUS OF CURVATURE.
      DIMENSION PA2(12,12),PA3(12,12,12),ELN(6,6)
```

```
      DIMENSION P33(9,9),P34(9,9),P35(9,9),P38(9,9),P39(9,9)
      DIMENSION P44(9,9),P45(9,9),P48(9,9),P49(9,9),P55(9,9)
      DIMENSION P58(9,9),P59(9,9),P88(9,9),P89(9,9),P99(9,9)
      COMMON /P13/ PA2,PA3
      COMMON /P14/ ELN,AREA,R
      COMMON /P19/ P33,P34,P35,P38,P39,P44,P45,P48,P49,P55
      COMMON /P20/ P58,P59,P88,P89,P99
      DO 1800 I=1,12
      DO 1800 J=1,12
      PA2(I,J)=0.0
      DO 1800 K=1,12
 1800 PA3(I,J,K)=0.0
C     COEFFICIENTS A(I,J),A(I,J,K),A(I,J,K,L) WHICH DEPENDS ON THE
C     MATERIAL PROPERTIES AND THE RADIUS OF CURVATURE.
      PA2(2,2)=ELN(1,1)
      PA2(2,3)=ELN(1,3)-0.5*ELN(1,6)/R*NFLAT
      PA2(2,5)=ELN(1,3)+1.5*ELN(1,6)/R*NFLAT
      PA2(2,6)=ELN(1,2)+ELN(1,5)/R*NFLAT
      PA2(2,7)=ELN(1,2)/R*NFLAT
      PA2(2,10)=-ELN(1,4)
      PA2(2,11)=-2.*ELN(1,6)
      PA2(2,12)=-ELN(1,5)
      PA2(3,3)=ELN(3,3)-0.5*ELN(3,6)/R*NFLAT+0.25*ELN(6,6)/(R*R)*NFLAT
      PA2(3,5)=ELN(3,3)+ELN(3,6)/R*NFLAT-0.75*ELN(6,6)/(R*R)*NFLAT
      PA2(3,6)=ELN(2,3)-0.5*ELN(2,6)/R*NFLAT+ELN(3,5)/R*NFLAT-0.5*
     1ELN(5,6)/(R*R)*NFLAT
      PA2(3,7)=ELN(2,3)/R*NFLAT-0.5*ELN(2,6)/(R*R)*NFLAT
      PA2(3,10)=-ELN(3,4)+0.5*ELN(4,6)/R*NFLAT
      PA2(3,11)=-2.*ELN(3,6)+ELN(6,6)/R*NFLAT
      PA2(3,12)=-ELN(3,5)+0.5*ELN(5,6)/R*NFLAT
      PA2(5,5)=ELN(3,3)+2.25*ELN(6,6)/(R*R)*NFLAT+1.5*ELN(3,6)/R*NFLAT
      PA2(5,6)=ELN(2,3)+1.5*ELN(2,6)/R*NFLAT+ELN(3,5)/R*NFLAT+1.5*
     1ELN(5,6)/(R*R)*NFLAT
      PA2(5,7)=ELN(2,3)/R*NFLAT+1.5*ELN(2,6)/(R*R)*NFLAT
      PA2(5,10)=-ELN(3,4)-1.5*ELN(4,6)/R*NFLAT
      PA2(5,11)=-2.*ELN(3,6)-3.*ELN(6,6)/R*NFLAT
      PA2(5,12)=-ELN(3,5)-1.5*ELN(5,6)/R*NFLAT
      PA2(6,6)=ELN(2,2)+2.*ELN(2,5)/R*NFLAT+ELN(5,5)/(R*R)*NFLAT
      PA2(6,7)=ELN(2,2)/R*NFLAT+ELN(2,5)/(R*R)*NFLAT
      PA2(6,10)=-ELN(2,4)-ELN(4,5)/R*NFLAT
      PA2(6,11)=-2.*ELN(2,6)-2.*ELN(5,6)/R*NFLAT
      PA2(6,12)=-ELN(2,5)-ELN(5,5)/R*NFLAT
      PA2(7,7)=ELN(2,2)/(R*R)*NFLAT
      PA2(7,10)=-ELN(2,4)/R*NFLAT
      PA2(7,11)=-2.*ELN(2,6)/R*NFLAT
      PA2(7,12)=-ELN(2,5)/R*NFLAT
      PA2(10,10)=ELN(4,4)
      PA2(10,11)=2.*ELN(4,6)
      PA2(10,12)=ELN(4,5)
      PA2(11,11)=4.*ELN(6,6)
      PA2(11,12)=2.*ELN(5,6)
      PA2(12,12)=ELN(5,5)
      DO 2001 K=1,11
      LL=K+1
```

```
      DO 2001 L=LL,12
 2001 PA2(L,K)=PA2(K,L)
C     COEFFICIENTS IN THE THIRD DEGREE.
      PA3(2,3,3)=(ELN(1,1)-ELN(1,2))/12.
      PA3(2,3,5)=(-ELN(1,1)+ELN(1,2))/12.
      PA3(2,4,4)=ELN(1,2)/(3.*R*R)*NFLAT
      PA3(2,4,8)=-ELN(1,3)/(3.*R)*NFLAT
      PA3(2,4,9)=-ELN(1,2)/(3.*R)*NFLAT
      PA3(2,5,5)=(ELN(1,1)-ELN(1,2))/12.
      PA3(2,8,8)=ELN(1,1)/3.
      PA3(2,8,9)=ELN(1,3)/3.
      PA3(2,9,9)=ELN(1,2)/3.
      PA3(3,3,3)=(ELN(1,3)-ELN(2,3))/4.+(-ELN(1,6)+ELN(2,6))/(8.*R)
     1*NFLAT
      PA3(3,3,5)=(-ELN(1,3)+ELN(2,3))/12.+5.*(ELN(1,6)-ELN(2,6))/
     1(24.*R)*NFLAT
      PA3(3,3,6)=(ELN(1,2)-ELN(2,2))/12.+(ELN(1,5)-ELN(2,5))/(12.*R)
     1*NFLAT
      PA3(3,3,7)=(ELN(1,2)-ELN(2,2))/(12.*R)*NFLAT
      PA3(3,3,10)=(-ELN(1,4)+ELN(2,4))/12.
      PA3(3,3,11)=(-ELN(1,6)+ELN(2,6))/6.
      PA3(3,3,12)=(-ELN(1,5)+ELN(2,5))/12.
      PA3(3,4,4)=ELN(2,3)/(3.*R*R)*NFLAT-ELN(2,6)/(6.*R**3)*NFLAT
      PA3(3,4,8)=-ELN(3,3)/(3.*R)*NFLAT+ELN(3,6)/(6.*R*R)*NFLAT
      PA3(3,4,9)=-ELN(2,3)/(3.*R)*NFLAT+ELN(2,6)/(6.*R*R)*NFLAT
      PA3(3,5,5)=(-ELN(1,3)+ELN(2,3))/12.+7.*(-ELN(1,6)+ELN(2,6))
     1/(24.*R)*NFLAT
      PA3(3,5,6)=(-ELN(1,2)+ELN(2,2))/12.+(-ELN(1,5)+ELN(2,5))/
     1(12.*R)*NFLAT
      PA3(3,5,7)=(-ELN(1,2)+ELN(2,2))/(12.*R)*NFLAT
      PA3(3,5,10)=(ELN(1,4)-ELN(2,4))/12.
      PA3(3,5,11)=(ELN(1,6)-ELN(2,6))/6.
      PA3(3,5,12)=(ELN(1,5)-ELN(2,5))/12.
      PA3(3,8,8)=ELN(1,3)/3.-ELN(1,6)/( 6.*R)*NFLAT
      PA3(3,8,9)=ELN(3,3)/3.-ELN(3,6)/(6.*R)*NFLAT
      PA3(3,9,9)=ELN(2,3)/3.-ELN(2,6)/(6.*R)*NFLAT
      PA3(4,4,5)=ELN(2,3)/(3.*R*R)*NFLAT+ELN(2,6)/(2.*R**3)*NFLAT
      PA3(4,4,6)=ELN(2,2)/(3.*R*R)*NFLAT+ELN(2,5)/(3.*R**3)*NFLAT
      PA3(4,4,7)=ELN(2,2)/(3.*R**3)*NFLAT
      PA3(4,4,10)=-ELN(2,4)/(3.*R*R)*NFLAT
      PA3(4,4,11)=-2.*ELN(2,6)/(3.*R*R)*NFLAT
      PA3(4,4,12)=-ELN(2,5)/(3.*R*R)*NFLAT
      PA3(4,5,8)=-ELN(3,3)/(3.*R)*NFLAT-ELN(3,6)/(2.*R*R)*NFLAT
      PA3(4,5,9)=-ELN(2,3)/(3.*R)*NFLAT-ELN(2,6)/(2.*R*R)*NFLAT
      PA3(4,6,8)=-ELN(2,3)/(3.*R)*NFLAT-ELN(3,5)/(3.*R*R)*NFLAT
      PA3(4,6,9)=-ELN(2,2)/(3.*R)*NFLAT-ELN(2,5)/(3.*R*R)*NFLAT
      PA3(4,7,8)=-ELN(2,3)/(3.*R*R)*NFLAT
      PA3(4,7,9)=-ELN(2,2)/(3.*R*R)*NFLAT
      PA3(4,8,10)=ELN(3,4)/(3.*R)*NFLAT
      PA3(4,8,11)=2.*ELN(3,6)/(3.*R)*NFLAT
      PA3(4,8,12)=ELN(3,5)/(3.*R)*NFLAT
      PA3(4,9,10)=ELN(2,4)/(3.*R)*NFLAT
      PA3(4,9,11)=2.*ELN(2,6)/(3.*R)*NFLAT
      PA3(4,9,12)=ELN(2,5)/(3.*R)*NFLAT
```

```
      PA3(5,5,5)=(ELN(1,3)-ELN(2,3))/4.+3.*(ELN(1,6)-ELN(2,6))/
     1(8.*R)*NFLAT
      PA3(5,5,6)=(ELN(1,2)-ELN(2,2))/12.+(ELN(1,5)-ELN(2,5))/
     1(12.*R)*NFLAT
      PA3(5,5,7)=(ELN(1,2)-ELN(2,2))/(12.*R)*NFLAT
      PA3(5,5,10)=(-ELN(1,4)+ELN(2,4))/12.
      PA3(5,5,11)=(-ELN(1,6)+ELN(2,6))/6.
      PA3(5,5,12)=(-ELN(1,5)+ELN(2,5))/12.
      PA3(5,8,8)=ELN(1,3)/3.+ELN(1,6)/(2.*R)*NFLAT
      PA3(5,8,9)=ELN(3,3)/3.+ELN(3,6)/(2.*R)*NFLAT
      PA3(5,9,9)=ELN(2,3)/3.+ELN(2,6)/(2.*R)*NFLAT
      PA3(6,8,8)=ELN(1,2)/3.+ELN(1,5)/(3.*R)*NFLAT
      PA3(6,8,9)=ELN(3,5)/(3.*R)*NFLAT+ELN(2,3)/3.
      PA3(6,9,9)=ELN(2,2)/3.+ELN(2,5)/(3.*R)*NFLAT
      PA3(7,8,8)=ELN(1,2)/(3.*R)*NFLAT
      PA3(7,8,9)=ELN(2,3)/(3.*R)*NFLAT
      PA3(7,9,9)=ELN(2,2)/(3.*R)*NFLAT
      PA3(8,8,10)=-ELN(1,4)/3.
      PA3(8,8,11)=-2.*ELN(1,6)/3.
      PA3(8,8,12)=-ELN(1,5)/3.
      PA3(8,9,10)=-ELN(3,4)/3.
      PA3(8,9,11)=-2.*ELN(3,6)/3.
      PA3(8,9,12)=-ELN(3,5)/3.
      PA3(9,9,10)=-ELN(2,4)/3.
      PA3(9,9,11)=-2.*ELN(2,6)/3.
      PA3(9,9,12)=-ELN(2,5)/3.
      DO 3000 I=1,12
      DO 3000 J=1,12
      DO 3000 K=1,12
      PA3(J,I,K)=PA3(I,J,K)
      PA3(I,K,J)=PA3(I,J,K)
 3000 CONTINUE
C     COEFFICIENTS IN THE FOURTH DEGREE EQUATION.
      DO 1 I=1,9
      DO 1 J=1,9
      P33(I,J)=0.0
      P34(I,J)=0.0
      P35(I,J)=0.0
      P38(I,J)=0.0
      P39(I,J)=0.0
      P44(I,J)=0.0
      P45(I,J)=0.0
      P48(I,J)=0.0
      P49(I,J)=0.0
      P55(I,J)=0.0
      P58(I,J)=0.0
      P59(I,J)=0.0
      P88(I,J)=0.0
      P89(I,J)=0.0
    1 P99(I,J)=0.0
      P33(8,8)=(ELN(1,1)-ELN(1,2))/48.
      P33(5,5)=3.*(ELN(1,1)-2.*ELN(1,2)+ELN(2,2))/192.
      P33(3,5)=(-ELN(1,1)+2.*ELN(1,2)-ELN(2,2))/64.
      P33(3,3)=(ELN(1,1)-2.*ELN(1,2)+ELN(2,2))/64.
```

```
P33(9,9)=(ELN(1,2)-ELN(2,2))/48.
P33(4,9)=(-ELN(1,2)+ELN(2,2))/(48.*R)*NFLAT
P33(4,4)=(ELN(1,2)-ELN(2,2))/(48.*R*R)*NFLAT
P33(8,9)=(ELN(1,3)-ELN(2,3))/48.
P33(4,8)=(-ELN(1,3)+ELN(2,3))/(48.*R)*NFLAT
P44(8,8)=(0.5*ELN(1,2)+ELN(3,3))/(6.*R*R)*NFLAT
P44(5,5)=(ELN(1,2)-ELN(2,2))/(48.*R*R)*NFLAT
P44(3,5)=(-ELN(1,2)+ELN(2,2))/(48.*R*R)*NFLAT
P44(3,3)=(ELN(1,2)-ELN(2,2))/(48.*R*R)*NFLAT
P44(9,9)=ELN(2,2)/(4.*R*R)*NFLAT
P44(4,9)=-ELN(2,2)/(4.*R**3)*NFLAT
P44(4,4)=ELN(2,2)/(4.*R**4)*NFLAT
P44(8,9)=ELN(2,3)/(4.*R*R)*NFLAT
P44(4,8)=-ELN(2,3)/(4.*R**3)*NFLAT
P55(8,8)=(ELN(1,1)-ELN(1,2))/48.
P55(5,5)=(ELN(1,1)-2.*ELN(1,2)+ELN(2,2))/64.
P55(3,5)=(-ELN(1,1)+2.*ELN(1,2)-ELN(2,2))/64.
P55(3,3)=3.*(ELN(1,1)-2.*ELN(1,2)+ELN(2,2))/192.
P55(9,9)=(ELN(1,2)-ELN(2,2))/48.
P55(4,9)=(-ELN(1,2)+ELN(2,2))/(48.*R)*NFLAT
P55(4,4)=(ELN(1,2)-ELN(2,2))/(48.*R*R)*NFLAT
P55(8,9)=(ELN(1,3)-ELN(2,3))/48.
P55(4,8)=(-ELN(1,3)+ELN(2,3))/(48.*R)*NFLAT
P88(8,8)=ELN(1,1)/4.
P88(5,5)=(ELN(1,1)-ELN(1,2))/48.
P88(3,5)=(-ELN(1,1)+ELN(1,2))/48.
P88(3,3)=(ELN(1,1)-ELN(1,2))/48.
P88(9,9)=ELN(1,2)/12.+ELN(3,3)/6.
P88(4,9)=-(ELN(1,2)+2.*ELN(3,3))/(12.*R)*NFLAT
P88(4,4)=(0.5*ELN(1,2)+ELN(3,3))/(6.*R*R)*NFLAT
P88(8,9)=ELN(1,3)/4.
P88(4,8)=-ELN(1,3)/(4.*R)*NFLAT
P99(8,8)=ELN(1,2)/12.+ELN(3,3)/6.
P99(5,5)=(ELN(1,2)-ELN(2,2))/48.
P99(3,5)=(-ELN(1,2)+ELN(2,2))/48.
P99(3,3)=(ELN(1,2)-ELN(2,2))/48.
P99(9,9)=ELN(2,2)/4.
P99(4,9)=-ELN(2,2)/(4.*R)*NFLAT
P99(4,4)=ELN(2,2)/(4.*R*R)*NFLAT
P99(8,9)=ELN(2,3)/4.
P99(4,8)=-ELN(2,3)/(4.*R)*NFLAT
P34(5,9)=(ELN(1,2)-ELN(2,2))/(48.*R)*NFLAT
P34(4,5)=(-ELN(1,2)+ELN(2,2))/48.
P34(3,9)=(-ELN(1,2)+ELN(2,2))/(48.*R)*NFLAT
P34(3,4)=(ELN(1,2)-ELN(2,2))/(48.*R*R)*NFLAT
P34(5,8)=(ELN(1,3)-ELN(2,3))/(48.*R)*NFLAT
P34(3,8)=(-ELN(1,3)+ELN(2,3))/(48.*R)*NFLAT
P35(8,8)=(-ELN(1,1)+ELN(1,2))/48.
P35(5,5)=(-ELN(1,1)+2.*ELN(1,2)-ELN(2,2))/4.
P35(3,5)=3.*(ELN(1,1)-2.*ELN(1,2)+ELN(2,2))/192.
P35(3,3)=(-ELN(1,1)+2.*ELN(1,2)-ELN(2,2))/64.
P35(9,9)=(-ELN(1,2)+ELN(2,2))/48.
P35(4,9)=(ELN(1,2)-ELN(2,2))/(48.*R)*NFLAT
P35(4,4)=(-ELN(1,2)+ELN(2,2))/(48.*R*R)*NFLAT
```

```
P35(8,9)=(-ELN(1,3)+ELN(2,3))/48.
P35(4,5)=(ELN(1,3)-ELN(2,3))/(48.*R)*NFLAT
P38(5,8)=(-ELN(1,1)+ELN(1,2))/48.
P38(3,8)=(ELN(1,1)-ELN(1,2))/48.
P38(5,9)=(-ELN(1,3)+ELN(2,3))/48.
P38(4,5)=(ELN(1,3)-ELN(2,3))/(48.*R)*NFLAT
P38(3,9)=(ELN(1,3)-ELN(2,3))/48.
P38(3,4)=(-ELN(1,3)+ELN(2,3))/(48.*R)*NFLAT
P39(5,9)=(-ELN(1,2)+ELN(2,2))/48.
P39(4,5)=(ELN(1,2)-ELN(2,2))/(48.*R)*NFLAT
P39(3,9)=(ELN(1,2)-ELN(2,2))/48.
P39(3,4)=(-ELN(1,2)+ELN(2,2))/(48.*R)*NFLAT
P39(5,8)=(-ELN(1,3)+ELN(2,3))/48.
P39(3,8)=(ELN(1,3)-ELN(2,3))/48.
P45(5,9)=(-ELN(1,2)+ELN(2,2))/(48.*R)*NFLAT
P45(4,5)=(ELN(1,2)-ELN(2,2))/(48.*R*R)*NFLAT
P45(3,9)=(ELN(1,2)-ELN(2,2))/(48.*R)*NFLAT
P45(3,4)=(-ELN(1,2)+ELN(2,2))/(48.*R*R)*NFLAT
P45(5,8)=(-ELN(1,3)+ELN(2,3))/(48.*R)*NFLAT
P45(3,8)=(ELN(1,3)-ELN(2,3))/(48.*R)*NFLAT
P48(8,9)=-(ELN(1,2)+2.*ELN(3,3))/(12.*R)*NFLAT
P48(4,8)=(0.5*ELN(1,2)+ELN(3,3))/(6.*R*R)*NFLAT
P48(8,8)=-ELN(1,3)/(4.*R)*NFLAT
P48(5,5)=(-ELN(1,3)+ELN(2,3))/(48.*R)*NFLAT
P48(3,5)=(ELN(1,3)-ELN(2,3))/(48.*R)*NFLAT
P48(3,3)=(-ELN(1,3)+ELN(2,3))/(48.*R)*NFLAT
P48(9,9)=-ELN(2,3)/(4.*R)*NFLAT
P48(4,9)=ELN(2,3)/(4.*R*R)*NFLAT
P48(4,4)=-ELN(2,3)/(4.*R**3)*NFLAT
P49(8,8)=-(ELN(1,2)+2.*ELN(3,3))/(12.*R)*NFLAT
P49(5,5)=(-ELN(1,2)+ELN(2,2))/(48.*R)*NFLAT
P49(3,5)=(ELN(1,2)-ELN(2,2))/(48.*R)*NFLAT
P49(3,3)=(-ELN(1,2)+ELN(2,2))/(48.*R)*NFLAT
P49(9,9)=-ELN(2,2)/(4.*R)*NFLAT
P49(4,9)=ELN(2,2)/(4.*R*R)*NFLAT
P49(4,4)=-ELN(2,2)/(4.*R**3)*NFLAT
P49(8,9)=-ELN(2,3)/(4.*R)*NFLAT
P49(4,8)=ELN(2,3)/(4.*R*R)*NFLAT
P58(5,8)=(ELN(1,1)-ELN(1,2))/48.
P58(3,8)=(-ELN(1,1)+ELN(1,2))/48.
P58(5,9)=(ELN(1,3)-ELN(2,3))/48.
P58(4,5)=(-ELN(1,3)+ELN(2,3))/(48.*R)*NFLAT
P58(3,9)=(-ELN(1,3)+ELN(2,3))/48.
P58(3,4)=(ELN(1,3)-ELN(2,3))/(48.*R)*NFLAT
P59(5,9)=(ELN(1,2)-ELN(2,2))/48.
P59(4,5)=(-ELN(1,2)+ELN(2,2))/(48.*R)*NFLAT
P59(3,9)=(-ELN(1,2)+ELN(2,2))/48.
P59(3,4)=(ELN(1,2)-ELN(2,2))/(48.*R)*NFLAT
P59(5,8)=(ELN(1,3)-ELN(2,3))/48.
P59(3,8)=(-ELN(1,3)+ELN(2,3))/48.
P89(8,9)=ELN(1,2)/12.+ELN(3,3)/6.
P89(4,8)=-(ELN(1,2)+2.*ELN(3,3))/(12.*R)*NFLAT
P89(8,8)=ELN(1,3)/4.
P89(5,5)=(ELN(1,3)-ELN(2,3))/48.
```

```
      P89(3,5)=(-ELN(1,3)+ELN(2,3))/48.
      P89(3,3)=(ELN(1,3)-ELN(2,3))/48.
      P89(9,9)=ELN(2,3)/4.
      P89(4,9)=-ELN(2,3)/(4.*R)*NFLAT
      P89(4,4)=ELN(2,3)/(4.*R*R)*NFLAT
      DO 2 I=1,8
      JL=I+1
      DO 2 J=JL,9
      P33(J,I)=P33(I,J)
      P44(J,I)=P44(I,J)
      P55(J,I)=P55(I,J)
      P88(J,I)=P88(I,J)
      P99(J,I)=P99(I,J)
      P34(J,I)=P34(I,J)
      P35(J,I)=P35(I,J)
      P38(J,I)=P38(I,J)
      P39(J,I)=P39(I,J)
      P45(J,I)=P45(I,J)
      P48(J,I)=P48(I,J)
      P49(J,I)=P49(I,J)
      P58(J,I)=P58(I,J)
      P59(J,I)=P59(I,J)
    2 P89(J,I)=P89(I,J)
      RETURN
      END
      SUBROUTINE FDIFF(NEL,NUMNP,NX1,NC2,WO,CONS1,CONS2,XO,YO,IEX,IPA3T,
     1NFLAT)
      DIMENSION NODE(244,9),ITYPE(244),X(240),Y(240),IBC(244,3)
      DIMENSION PA2(12,12),PA3(12,12,12),NODES(244)
      DIMENSION QK(27,27),DQK(27,27),RE(27),B(6,9),CA2(27,27)
      COMMON /P15/ X,Y,NODE,ITYPE,IBC,NODES
      COMMON /P13/ PA2,PA3
      COMMON /P11/ QK,DQK,RE,B,CA2
      DO 10 M=1,NEL
      L1=NODE(M,1)
      L2=NODE(M,2)
      L3=NODE(M,3)
      L4=NODE(M,4)
      L5=NODE(M,5)
      XH=X(L2)-X(L3)
      XK=X(L1)-X(L2)
      XL=Y(L2)-Y(L5)
      XM=Y(L4)-Y(L2)
      IEL=ITYPE(M)
      GO TO (1,2,3,4,5,6,7,8,9),IEL
C     BOUNDARY ELEMENT ITYPE(1)=1--LEFT EXTERIOR BOUNDARY ELEMENT
    1 AREA=(XL+XM)*XH/4.0
      X1=((XM-XL)*(3.*XL+XM))/(16.*XM*(XL+XM))
      X2=((3.*XM+XL)*(3.*XL+XM))/(16.*XM*XL)
      X3=-((XM-XL)*(3.*XM+XL))/(16.*XL*(XL+XM))
      Y1=-(3.*XH*XH)/(16.*XK*(XH+XK))
      Y2=0.25*(XK+0.75*XH)/XK
      Y3=0.75*(XK+0.75*XH)/(XH+XK)
      XC=X(L3)+XH/4.
```

```
YC=Y(L5)+(3.*XL+XM)/4.
B(1,1)=X2*Y1
B(1,2)=X2*Y2
B(1,3)=X2*Y3
B(1,4)=X1*Y2
B(1,5)=X3*Y2
B(1,6)=X3*Y3
B(1,7)=X1*Y3
B(1,8)=X1*Y1
B(1,9)=X3*Y1
Z1=-XH/(2.*XK*(XH+XK))
Z2=(XK+0.5*XH)/(XH*XK)
Z3=-(XK+1.5*XH)/(XH*(XH+XK))
B(2,1)=X2*Z1
B(2,2)=X2*Z2
B(2,3)=X2*Z3
B(2,4)=X1*Z2
B(2,5)=X3*Z2
B(2,6)=X3*Z3
B(2,7)=X1*Z3
B(2,8)=X1*Z1
B(2,9)=X3*Z1
B(3,1)=Y1*(1./(2.*XL)-1./(2.*XM))
B(3,2)=Y2*(1./(2.*XL)-1./(2.*XM))
B(3,3)=Y3*(1./(2.*XL)-1./(2.*XM))
B(3,4)=Y2/(2.*XM)
B(3,5)=-Y2/(2.*XL)
B(3,6)=-Y3/(2.*XL)
B(3,7)=Y3/(2.*XM)
B(3,8)=Y1/(2.*XM)
B(3,9)=-Y1/(2.*XL)
B(4,1)=2.*X2/(XK*(XH+XK))
B(4,2)=-2.*X2/(XK*XH)
B(4,3)=2.*X2/(XH*(XH+XK))
B(4,4)=-2.*X1/(XK*XH)
B(4,5)=-2.*X3/(XK*XH)
B(4,6)=2.*X3/(XH*(XH+XK))
B(4,7)=2.*X1/(XH*(XH+XK))
B(4,8)=2.*X1/(XK*(XH+XK))
B(4,9)=2.*X3/(XK*(XH+XK))
B(5,1)=Z1*(1./(2.*XL)-1./(2.*XM))
B(5,2)=Z2*(1./(2.*XL)-1./(2.*XM))
B(5,3)=Z3*(1./(2.*XL)-1./(2.*XM))
B(5,4)=Z2/(2.*XM)
B(5,5)=-Z2/(2.*XL)
B(5,6)=-Z3/(2.*XL)
B(5,7)=Z3/(2.*XM)
B(5,8)=Z1/(2.*XM)
B(5,9)=-Z1/(2.*XL)
B(6,1)=-2.*Y1/(XL*XM)
B(6,2)=-2.*Y2/(XL*XM)
B(6,3)=-2.*Y3/(XL*XM)
B(6,4)=2.*Y2/(XM*(XL+XM))
B(6,5)=2.*Y2/(XL*(XL+XM))
```

```
      B(6,6)=2.*Y3/(XL*(XL+XM))
      B(6,7)=2.*Y3/(XM*(XL+XM))
      B(6,8)=2.*Y1/(XM*(XL+XM))
      B(6,9)=2.*Y1/(XL*(XL+XM))
C     CALCULATE THE B COEFFICIENTS FOR THE FINITE DIFFERENCE FORMULAS
C     AT THE CENTROID OF THE ELEMENT AREA.
C     AT THIS POINT ELEMENT AREA, MATRIX T AND MATRIX B ARE AVAILABLE
C     FOR ELEMENT M.
      GO TO 70
C     BOUNDARY ELEMENT ITYPE(2)=2--TOP EXTERIOR ELEMENT
    2 AREA=(XH+XK)*XM/4.0
      X1=((XK-XH)*(3.*XH+XK))/(16.*XK*(XH+XK))
      X2=((3.*XK+XH)*(3.*XH+XK))/(16.*XH*XK)
      X3=-((XK-XH)*(3.*XK+XH))/(16.*XH*(XH+XK))
      Y1=-(3.*XM*XM)/(16.*XL*(XM+XL))
      Y2=(0.75*XM+XL)/(4.*XL)
      Y3=0.75*(0.75*XM+XL)/(XM+XL)
      XC=X(L3)+(3.*XH+XK)/4.
      YC=Y(L4)-XM/4.
      B(1,1)=X1*Y2
      B(1,2)=X2*Y2
      B(1,3)=X3*Y2
      B(1,4)=X2*Y3
      B(1,5)=X2*Y1
      B(1,6)=X3*Y1
      B(1,7)=X3*Y3
      B(1,8)=X1*Y3
      B(1,9)=X1*Y1
      B(2,1)=Y2/(2.*XK)
      B(2,2)=Y2*(1./(2.*XH)-1./(2.*XK))
      B(2,3)=-Y2/(2.*XH)
      B(2,4)=Y3*(1./(2.*XH)-1./(2.*XK))
      B(2,5)=Y1*(1./(2.*XH)-1./(2.*XK))
      B(2,6)=-Y1/(2.*XH)
      B(2,7)=-Y3/(2.*XH)
      B(2,8)=Y3/(2.*XK)
      B(2,9)=Y1/(2.*XK)
      Z1=XM/(2.*XL*(XM+XL))
      Z2=-(0.5*XM+XL)/(XM*XL)
      Z3=(1.5*XM+XL)/(XM*(XM+XL))
      B(3,1)=X1*Z2
      B(3,2)=X2*Z2
      B(3,3)=X3*Z2
      B(3,4)=X2*Z3
      B(3,5)=X2*Z1
      B(3,6)=X3*Z1
      B(3,7)=X3*Z3
      B(3,8)=X1*Z3
      B(3,9)=X1*Z1
      B(4,1)=2.*Y2/(XK*(XH+XK))
      B(4,2)=-2.*Y2/(XK*XH)
      B(4,3)=2.*Y2/(XH*(XH+XK))
      B(4,4)=-2.*Y3/(XH*XK)
      B(4,5)=-2.*Y1/(XH*XK)
```

```
      B(4,6)=2.*Y1/(XH*(XH+XK))
      B(4,7)=2.*Y3/(XH*(XH+XK))
      B(4,8)=2.*Y3/(XK*(XH+XK))
      B(4,9)=2.*Y1/(XK*(XH+XK))
      B(5,1)=Z2/(2.*XK)
      B(5,2)=Z2*(1./(2.*XH)-1./(2.*XK))
      B(5,3)=-Z2/(2.*XH)
      B(5,4)=Z3*(1./(2.*XH)-1./(2.*XK))
      B(5,5)=Z1*(1./(2.*XH)-1./(2.*XK))
      B(5,6)=-Z1/(2.*XH)
      B(5,7)=-Z3/(2.*XH)
      B(5,8)=Z3/(2.*XK)
      B(5,9)=Z1/(2.*XK)
      B(6,1)=-2.*X1/(XL*XM)
      B(6,2)=-2.*X2/(XL*XM)
      B(6,3)=-2.*X3/(XL*XM)
      B(6,4)=2.*X2/(XM*(XL+XM))
      B(6,5)=2.*X2/(XL*(XL+XM))
      B(6,6)=2.*X3/(XL*(XL+XM))
      B(6,7)=2.*X3/(XM*(XL+XM))
      B(6,8)=2.*X1/(XM*(XL+XM))
      B(6,9)=2.*X1/(XL*(XL+XM))
C     CALCULATE THE B COEFFICIENTS FOR THE FINITE DIFFERENCE FORMULAS
C     AT THE CENTROID OF THE ELEMENT AREA
C     AT THIS POINT THE ELEMENT AREA, MATRIX T AND MATRIX B ARE
C     AVAILABLE FOR THE ELEMENT M.
      GO TO 70
C     BOUNDARY ELEMENT ITYPE(3)=3---RIGHT EXTERIOR BOUNDARY ELEMENT
    3 AREA=(XL+XM)*XK/4.0
      X1=((XM-XL)*(3.*XL+XM))/(16.*XM*(XL+XM))
      X2=((3.*XM+XL)*(3.*XL+XM))/(16.*XL*XM)
      X3=-((XM-XL)*(3.*XM+XL))/(16.*XL*(XL+XM))
      Y1=(0.75*(XH+0.75*XK))/(XH+XK)
      Y2=(0.25*(XH+0.75*XK))/XH
      Y3=-(3.*XK*XK)/(16.*XH*(XH+XK))
      XC=X(L1)-XK/4.
      YC=Y(L5)+(3.*XL+XM)/4.
      B(1,1)=X2*Y1
      B(1,2)=X2*Y2
      B(1,3)=X2*Y3
      B(1,4)=X1*Y2
      B(1,5)=X3*Y2
      B(1,6)=X3*Y3
      B(1,7)=X1*Y3
      B(1,8)=X1*Y1
      B(1,9)=X3*Y1
      B(2,1)=X2*(XH+1.5*XK)/(XK*(XH+XK))
      B(2,2)=-X2*(XH+0.5*XK)/(XH*XK)
      B(2,3)=X2*XK/(2.*XH*(XH+XK))
      B(2,4)=-X1*(XH+0.5*XK)/(XH*XK)
      B(2,5)=-X3*(XH+0.5*XK)/(XH*XK)
      B(2,6)=X3*XK/(2.*XH*(XH+XK))
      B(2,7)=X1*XK/(2.*XH*(XH+XK))
      B(2,8)=X1*(XH+1.5*XK)/(XK*(XH+XK))
```

```
      B(2,9)=X3*(XH+1.5*XK)/(XK*(XH+XK))
      B(3,1)=Y1*(1./(2.*XL)-1./(2.*XM))
      B(3,2)=Y2*(1./(2.*XL)-1./(2.*XM))
      B(3,3)=Y3*(1./(2.*XL)-1./(2.*XM))
      B(3,4)=Y2/(2.*XM)
      B(3,5)=-Y2/(2.*XL)
      B(3,6)=-Y3/(2.*XL)
      B(3,7)=Y3/(2.*XM)
      B(3,8)=Y1/(2.*XM)
      B(3,9)=-Y1/(2.*XL)
      B(4,1)=(2.*X2)/(XK*(XH+XK))
      B(4,2)=-2.*X2/(XH*XK)
      B(4,3)=(2.*X2)/(XH*(XH+XK))
      B(4,4)=-2.*X1/(XH*XK)
      B(4,5)=-2.*X3/(XH*XK)
      B(4,6)=2.*X3/(XH*(XH+XK))
      B(4,7)=2.*X1/(XH*(XH+XK))
      B(4,8)=2.*X1/(XK*(XH+XK))
      B(4,9)=2.*X3/(XK*(XH+XK))
      Z1=(XH+1.5*XK)/(XK*(XH+XK))
      Z2=-(XH+0.5*XK)/(XK*XH)
      Z3=XK/(2.*XH*(XH+XK))
      B(5,1)=Z1*(1./(2.*XL)-1./(2.*XM))
      B(5,2)=Z2*(1./(2.*XL)-1./(2.*XM))
      B(5,3)=Z3*(1./(2.*XL)-1./(2.*XM))
      B(5,4)=Z2/(2.*XM)
      B(5,5)=-Z2/(2.*XL)
      B(5,6)=-Z3/(2.*XL)
      B(5,7)=Z3/(2.*XM)
      B(5,8)=Z1/(2.*XM)
      B(5,9)=-Z1/(2.*XL)
      B(6,1)=-2.*Y1/(XL*XM)
      B(6,2)=-2.*Y2/(XL*XM)
      B(6,3)=-2.*Y3/(XL*XM)
      B(6,4)=2.*Y2/(XM*(XL+XM))
      B(6,5)=2.*Y2/(XL*(XL+XM))
      B(6,6)=2.*Y3/(XL*(XL+XM))
      B(6,7)=2.*Y3/(XM*(XL+XM))
      B(6,8)=2.*Y1/(XM*(XL+XM))
      B(6,9)=2.*Y1/(XL*(XL+XM))
C     CALCULATE THE B COEFFICIENTS FOR THE FINITE DIFFERENCE FORMULAS
C     AT THE CENTROID OF THE ELEMENT AREA.0
C     AT THIS POINT THE ELEMENT AREA, THE MATRIX T AND THE MATRIX B
C     ARE AVAILABLE FOR THE ELEMENT M.0
      GO TO 70
C     BOUNDARY ELEMENT ITYPE(4)=4--BOTTOM EXTERIOR BOUNDARY ELEMENT
    4 AREA=(XH+XK)*XL/4.0
      X1=-3.*XL*XL/(16.*XM*(XM+XL))
      X2=(XM+0.75*XL)/(4.*XM)
      X3=0.75*(XM+0.75*XL)/(XM+XL)
      Y1=((XK-XH)*(3.*XH+XK))/(16.*XK*(XH+XK))
      Y2=((XH+3.*XK)*(3.*XH+XK))/(16.*XH*XK)
      Y3=-((XK-XH)*(3.*XK+XH))/(16.*XH*(XH+XK))
      Z1=-XL/(2.*XM*(XM+XL))
```

```
      Z2=(XM+0.5*XL)/(XM*XL)
      Z3=-(XM+1.5*XL)/(XL*(XM+XL))
      XC=X(L3)+(3.*XH+XK)/4.
      YC=Y(L5)-XL/4.
      B(1,1)=X2*Y1
      B(1,2)=X2*Y2
      B(1,3)=X2*Y3
      B(1,4)=X1*Y2
      B(1,5)=X3*Y2
      B(1,6)=X3*Y3
      B(1,7)=X1*Y3
      B(1,8)=X1*Y1
      B(1,9)=X3*Y1
      B(2,1)=X2/(2.*XK)
      B(2,2)=X2*(1./(2.*XH)-1./(2.*XK))
      B(2,3)=-X2/(2.*XH)
      B(2,4)=X1*(1./(2.*XH)-1./(2.*XK))
      B(2,5)=X3*(1./(2.*XH)-1./(2.*XK))
      B(2,6)=-X3/(2.*XH)
      B(2,7)=-X1/(2.*XH)
      B(2,8)=X1/(2.*XK)
      B(2,9)=X3/(2.*XK)
      B(3,1)=Y1*Z2
      B(3,2)=Y2*Z2
      B(3,3)=Y3*Z2
      B(3,4)= Y2*Z1
      B(3,5)= Y2*Z3
      B(3,6)= Y3*Z3
      B(3,7)= Y3*Z1
      B(3,8)= Y1*Z1
      B(3,9)= Y1*Z3
      B(4,1)=2.*X2/(XK*(XH+XK))
      B(4,2)=-2.*X2/(XK*XH)
      B(4,3)=2.*X2/(XH*(XH+XK))
      B(4,4)=-2.*X1/(XK*XH)
      B(4,5)=-2.*X3/(XK*XH)
      B(4,6)=2.*X3/(XH*(XH+XK))
      B(4,7)=2.*X1/(XH*(XH+XK))
      B(4,8)=2.*X1/(XK*(XH+XK))
      B(4,9)=2.*X3/(XK*(XH+XK))
      B(5,1)=Z2/(2.*XK)
      B(5,2)=Z2*(1./XH-1./XK)/2.
      B(5,3)=-Z2/(2.*XH)
      B(5,4)=Z1*(1./XH-1./XK)/2.
      B(5,5)=Z3*(1./XH-1./XK)/2.
      B(5,6)=-Z3/(2.*XH)
      B(5,7)=-Z1/(2.*XH)
      B(5,8)=Z1/(2.*XK)
      B(5,9)=Z3/(2.*XK)
      B(6,1)=-2.*Y1/(XL*XM)
      B(6,2)=-2.*Y2/(XL*XM)
      B(6,3)=-2.*Y3/(XL*XM)
      B(6,4)=2.*Y2/(XM*(XM+XL))
      B(6,5)=2.*Y2/(XL*(XM+XL))
```

```
      B(6,6)=2.*Y3/(XL*(XM+XL))
      B(6,7)=2.*Y3/(XM*(XM+XL))
      B(6,8)=2.*Y1/(XM*(XM+XI))
      B(6,9)=2.*Y1/(XL*(XM+XL))
C     CALCULATE THE B COEFFICIENTS FOR THE FINITE DIFFERENCE FORMULAS
C     AT THE CENTROID OF THE ELEMENT AREA
C     AT THIS POINT THE ELEMENT AREA,THE MATRIX T AND THE MATRIX B ARE
C     AVAILABLE FOR THE ELEMENT M.
      GO TO 70
C     INTERIOR ELEMENT ITYPE(5)=5
    5 AREA=(XH+XK)*(XL+XM)/4.0
      X1=((XM-XL)*(3.*XL+XM))/(16.*XM*(XL+XM))
      X2=((3.*XM+XL)*(3.*XI+XM))/(16.*XL*XM)
      X3=-((XM-XL)*(3.*XM+XL))/(16.*XL*(XM+XL))
      Y1=-((XK-XH)*(3.*XK+XH))/(16.*XH*(XH+XK))
      Y2=((3.*XH+XK)*(3.*XK+XH))/(16.*XH*XK)
      Y3=((XK-XH)*(3.*XH+XK))/(16.*XK*(XH+XK))
      XC=X(L3)+(3.*XH+XK)/4.
      YC=Y(L5)+(3.*XL+XM)/4.
      B(1,1)=X2*Y3
      B(1,2)=X2*Y2
      B(1,3)=X2*Y1
      B(1,4)=X1*Y2
      B(1,5)=X3*Y2
      B(1,6)=X3*Y1
      B(1,7)=X1*Y1
      B(1,8)=X1*Y3
      B(1,9)=X3*Y3
      X4=-1./(2.*XH)
      X5=1./(2.*XH)-1./(2.*XK)
      X6=1./(2.*XK)
      Y4=-1./(2.*XL)
      Y5=1./(2.*XL)-1./(2.*XM)
      Y6=1./(2.*XM)
      B(2,1)=X2*X6
      B(2,2)=X2*X5
      B(2,3)=X2*X4
      B(2,4)=X1*X5
      B(2,5)=X3*X5
      B(2,6)=X3*X4
      B(2,7)=X1*X4
      B(2,8)=X1*X6
      B(2,9)=X3*X6
      B(3,1)=Y3*Y5
      B(3,2)=Y2*Y5
      B(3,3)=Y1*Y5
      B(3,4)=Y2*Y6
      B(3,5)=Y2*Y4
      B(3,6)=Y1*Y4
      B(3,7)=Y1*Y6
      B(3,8)=Y3*Y6
      B(3,9)=Y3*Y4
      Z1=2./(XK*(XH+XK))
      Z2=-2./(XH*XK)
```

```
      Z3=2./(XH*(XH+XK))
      Z4=2./(XM*(XM+XL))
      Z5=-2./(XL*XM)
      Z6=2./(XL*(XM+XL))
      B(4,1)=X2*Z1
      B(4,2)=X2*Z2
      B(4,3)=X2*Z3
      B(4,4)=X1*Z2
      B(4,5)=X3*Z2
      B(4,6)=X3*Z3
      B(4,7)=X1*Z3
      B(4,8)=X1*Z1
      B(4,9)=X3*Z1
      B(5,1)=Y5*X6
      B(5,2)=Y5*X5
      B(5,3)=Y5*X4
      B(5,4)=Y6*X5
      B(5,5)=Y4*X5
      B(5,6)=Y4*X4
      B(5,7)=Y6*X4
      B(5,8)=Y6*X6
      B(5,9)=Y4*X6
      B(6,1)=Y3*Z5
      B(6,2)=Y2*Z5
      B(6,3)=Y1*Z5
      B(6,4)=Y2*Z4
      B(6,5)=Y2*Z6
      B(6,6)=Y1*Z6
      B(6,7)=Y1*Z4
      B(6,8)=Y3*Z4
      B(6,9)=Y3*Z6
      GO TO 70
C     ITYPE(6)   -   UPPER LEFT CORNER ELEMENT.
    6 AREA=(XH*XM)/4.
      XC=X(L4)-0.75*XH
      YC=Y(L3)+0.75*XM
      X1=(-3.*XM*XM)/(16.*XL*(XL+XM))
      X2=(XL+0.75*XM)/(4.*XL)
      X3=0.75*(XL+0.75*XM)/(XM+XL)
      Y1=(-3.*XH*XH)/(16.*XK*(XH+XK))
      Y2=(XK+0.75*XH)/(4.*XK)
      Y3=0.75*(XK+0.75*XH)/(XH+XK)
      B(1,1)=X2*Y1
      B(1,2)=X2*Y2
      B(1,3)=X2*Y3
      B(1,4)=X3*Y2
      B(1,5)=X1*Y2
      B(1,6)=X1*Y3
      B(1,7)=X3*Y3
      B(1,8)=X3*Y1
      B(1,9)=X1*Y1
      Z1=-XH/(2.*XK*(XH+XK))
      Z2=(XK+0.5*XH)/(XH*XK)
      Z3=-(XK+1.5*XH)/(XH*(XH+XK))
```

```
B(2,1)=X2*Z1
B(2,2)=X2*Z2
B(2,3)=X2*Z3
B(2,4)=X3*Z2
B(2,5)=X1*Z2
B(2,6)=X1*Z3
B(2,7)=X3*Z3
B(2,8)=X3*Z1
B(2,9)=X1*Z1
ZZ1=XM/(2.*XL*(XL+XM))
ZZ2=-(XL+0.5*XM)/(XL*XM)
ZZ3=(XL+1.5*XM)/(XM*(XL+XM))
B(3,1)=Y1*ZZ2
B(3,2)=Y2*ZZ2
B(3,3)=Y3*ZZ2
B(3,4)=Y2*ZZ3
B(3,5)=Y2*ZZ1
B(3,6)=Y3*ZZ1
B(3,7)=Y3*ZZ3
B(3,8)=Y1*ZZ3
B(3,9)=Y1*ZZ1
ZZ4=2./(XK*(XH+XK))
ZZ5=-2./(XH*XK)
ZZ6=2./(XH*(XH+XK))
B(4,1)=X2*ZZ4
B(4,2)=X2*ZZ5
B(4,3)=X2*ZZ6
B(4,4)=X3*ZZ5
B(4,5)=X1*ZZ5
B(4,6)=X1*ZZ6
B(4,7)=X3*ZZ6
B(4,8)=X3*ZZ4
B(4,9)=X1*ZZ4
B(5,1)=ZZ2*Z1
B(5,2)=ZZ2*Z2
B(5,3)=ZZ2*Z3
B(5,4)=ZZ3*Z2
B(5,5)=ZZ1*Z2
B(5,6)=ZZ1*Z3
B(5,7)=ZZ3*Z3
B(5,8)=ZZ3*Z1
B(5,9)=ZZ1*Z1
Y4=2./(XL*(XL+XM))
Y5=-2./(XL*XM)
Y6=2./(XM*(XL+XM))
B(6,1)=Y1*Y5
B(6,2)=Y2*Y5
B(6,3)=Y3*Y5
B(6,4)=Y2*Y6
B(6,5)=Y2*Y4
B(6,6)=Y3*Y4
B(6,7)=Y3*Y6
B(6,8)=Y1*Y6
B(6,9)=Y1*Y4
```

```
      GO TO 70
C     ITYPE(7)  -  UPPER RIGHT CORNER ELEMENT.
    7 AREA=(XK*XM)/4.
      XC=X(L2)+0.75*XK
      YC=Y(L1)+0.75*XM
      X1=(-3.*XM*XM)/(16.*XL*(XL+XM))
      X2=0.25*(XL+0.75*XM)/XL
      X3=0.75*(XL+0.75*XM)/(XL+XM)
      Y1=(-3.*XK*XK)/(16.*XH*(XH+XK))
      Y2=0.25*(XH+0.75*XK)/XH
      Y3=0.75*(XH+0.75*XK)/(XH+XK)
      Y4=XK/(2.*XH*(XH+XK))
      Y5=-(XH+0.5*XK)/(XH*XK)
      Y6=(XH+1.5*XK)/(XH+XK)
      B(1,1)=X2*Y3
      B(1,2)=X2*Y2
      B(1,3)=X2*Y1
      B(1,4)=X3*Y2
      B(1,5)=X1*Y2
      B(1,6)=X1*Y1
      B(1,7)=X3*Y1
      B(1,8)=X3*Y3
      B(1,9)=X1*Y3
      B(2,1)=X2*Y6
      B(2,2)=X2*Y5
      B(2,3)=X2*Y4
      B(2,4)=X3*Y5
      B(2,5)=X1*Y5
      B(2,6)=X1*Y4
      B(2,7)=X3*Y4
      B(2,8)=X3*Y6
      B(2,9)=X1*Y6
      X4=XM/(2.*XL*(XL+XM))
      X5=-(XL+0.5*XM)/(XL*XM)
      X6=(XL+1.5*XM)/(XM*(XL+XM))
      B(3,1)=Y3*X5
      B(3,2)=Y2*X5
      B(3,3)=Y1*X5
      B(3,4)=Y2*X6
      B(3,5)=Y2*X4
      B(3,6)=Y1*X4
      B(3,7)=Y1*X6
      B(3,8)=Y3*X6
      B(3,9)=Y3*X4
      Y7=2./(XH*(XH+XK))
      Y8=-2./(XH*XK)
      Y9=2./(XK*(XH+XK))
      B(4,1)=X2*Y9
      B(4,2)=X2*Y8
      B(4,3)=X2*Y7
      B(4,4)=X3*Y8
      B(4,5)=X1*Y8
      B(4,6)=X1*Y7
      B(4,7)=X3*Y7
```

```
      B(4,8)=X3*Y9
      B(4,9)=X1*Y9
      B(5,1)=X5*Y6
      B(5,2)=X5*Y5
      B(5,3)=X5*Y4
      B(5,4)=X6*Y5
      B(5,5)=X4*Y5
      B(5,6)=X4*Y4
      B(5,7)=X6*Y4
      B(5,8)=X6*Y6
      B(5,9)=X4*Y6
      X7=2./(XL*(XL+XM))                                                00000
      X8=-2./(XL*XM)                                                    00000
      X9=2./(XM*(XL+XM))                                                00000
      B(6,1)=Y3*X8                                                      00000
      B(6,2)=Y2*X8                                                      00000
      B(6,3)=Y1*X8                                                      00000
      B(6,4)=Y2*X9                                                      00000
      B(6,5)=Y2*X7                                                      00000
      B(6,6)=Y1*X7                                                      00000
      B(6,7)=Y1*X9                                                      00000
      B(6,8)=Y3*X9                                                      00000
      B(6,9)=Y3*X7                                                      00000
      GO TO 70                                                          00000
C     ITYPE(8)   - LOWER LEFT CORNER ELEMENT.                           00000
    8 AREA=(XH*XL)/4.
      XC=X(L3)+0.25*XH
      YC=Y(L3)-0.75*XL
      X1=(-3.*XL*XL)/(16.*XM*(XM+XL))                                   00000
      X2=0.25*(XM+0.75*XL)/XM                                           00000
      X3=0.75*(XM+0.75*XL)/(XM+XL)                                      00000
      Y1=(-3.*XH*XH)/(16.*XK*(XH+XK))                                   00000
      Y2=0.25*(XK+0.75*XH)/XK                                           00000
      Y3=0.75*(XK+0.75*XH)/(XH+XK)                                      00000
      Y4=-XH/(2.*XK*(XH+XK))                                            00000
      Y5=(XK+0.5*XH)/(XH*XK)                                            00000
      Y6=-(XK+1.5*XH)/(XH*(XH+XK))                                      00000
      Y7=2./(XK*(XH+XK))                                                00000
      Y8=-2./(XH*XK)                                                    00000
      Y9=2./(XH*(XH+XK))                                                00000
      X4=-XL/(2.*XM*(XM+XL))                                            00000
      X5=(XM+0.5*XL)/(XL*XM)                                            00000
      X6=-(XM+1.5*XL)/(XL*(XL+XM))                                      00000
      X7=2./(XM*(XM+XL))                                                00000
      X8=-2./(XL*XM)                                                    00000
      X9=2./(XL*(XM+XL))                                                00000
      B(1,1)=X2*Y1                                                      00000
      B(1,2)=X2*Y2                                                      00000
      B(1,3)=X2*Y3                                                      00000
      B(1,4)=X1*Y2                                                      00000
      B(1,5)=X3*Y2                                                      00000
      B(1,6)=X3*Y3                                                      00000
      B(1,7)=X1*Y3                                                      00000
      B(1,8)=X1*Y1                                                      00000
```

```
      B(1,9)=X3*Y1                                                      00000
      B(2,1)=X2*Y4                                                      00000
      B(2,2)=X2*Y5                                                      00000
      B(2,3)=X2*Y6                                                      00000
      B(2,4)=X1*Y5                                                      00000
      B(2,5)=X3*Y5                                                      00000
      B(2,6)=X3*Y6                                                      00000
      B(2,7)=X1*Y6                                                      00000
      B(2,8)=X1*Y4                                                      00000
      B(2,9)=X3*Y4                                                      00000
      B(3,1)=Y1*X5                                                      00000
      B(3,2)=Y2*X5                                                      00000
      B(3,3)=Y3*X5                                                      00000
      B(3,4)=Y2*X4                                                      00000
      B(3,5)=Y2*X6                                                      00000
      B(3,6)=Y3*X6                                                      00000
      B(3,7)=Y3*X4                                                      00000
      B(3,8)=Y1*X4                                                      00000
      B(3,9)=Y1*X6                                                      00000
      B(4,1)=X2*Y7                                                      00000
      B(4,2)=X2*Y8                                                      00000
      B(4,3)=X2*Y9                                                      00000
      B(4,4)=X1*Y8                                                      00000
      B(4,5)=X3*Y8                                                      00000
      B(4,6)=X3*Y9                                                      00000
      B(4,7)=X1*Y9                                                      00000
      B(4,8)=X1*Y7                                                      00000
      B(4,9)=X3*Y7                                                      00000
      B(5,1)=Y4*X5                                                      00000
      B(5,2)=Y5*X5                                                      00000
      B(5,3)=Y6*X5                                                      00000
      B(5,4)=Y5*X4                                                      00000
      B(5,5)=Y5*X6                                                      00000
      B(5,6)=Y6*X6                                                      00000
      B(5,7)=Y6*X4                                                      00000
      B(5,8)=Y4*X4                                                      00000
      B(5,9)=Y4*X6                                                      00000
      B(6,1)=Y1*X8                                                      00000
      B(6,2)=Y2*X8                                                      00000
      B(6,3)=Y3*X8                                                      00000
      B(6,4)=Y2*X7                                                      00000
      B(6,5)=Y2*X9                                                      00000
      B(6,6)=Y3*X9                                                      00000
      B(6,7)=Y3*X7                                                      00000
      B(6,8)=Y1*X7                                                      00000
      B(6,9)=Y1*X9                                                      00000
      GO TO 70                                                          00000
C     ITYPE(9)   -   LOWER RIGHT CORNER ELEMENT.                        00000
    9 AREA=(XL*XK)/4.
      XC=X(L5)+0.75*XK
      YC=Y(L5)+0.25*XL
      X1=(-3.*XL*XL)/(16.*XM*(XM+XL))                                   00000
      X2=0.25*(XM+0.75*XL)/XM                                           00000
      X3=0.75*(XM+0.75*XL)/(XM+XL)                                      00000
```

```
      Y1=(-3.*XK*XK)/(16.*XH*(XH+XK))                                  00000
      Y2=0.25*(XH+0.75*XK)/XH                                          00001
      Y3=0.75*(XH+0.75*XK)/(XH+XK)                                     00001
      Y4=XK/(2.*XH*(XH+XK))                                            00001
      Y5=-(XH+0.5*XK)/(XH*XK)                                          00001
      Y6=(XH+1.5*XK)/(XK*(XH+XK))                                      00001
      Y7=2./(XH*(XH+XK))                                               00001
      Y8=-2./(XH*XK)                                                   00001
      Y9=2./(XK*(XH+XK))                                               00001
      X4=-XL/(2.*XM*(XM+XL))                                           00001
      X5=(XM+0.5*XL)/(XL*XM)                                           00001
      X6=-(XM+1.5*XL)/(XL*(XL+XM))                                     00001
      X7=2./(XM*(XM+XL))                                               00001
      X8=-2./(XL*XM)                                                   00001
      X9=2./(XL*(XM+XL))                                               00001
      B(1,1)=X2*Y3                                                     00001
      B(1,2)=X2*Y2                                                     00001
      B(1,3)=X2*Y1                                                     00001
      B(1,4)=X1*Y2                                                     00001
      B(1,5)=X3*Y2                                                     00001
      B(1,6)=X3*Y1                                                     00001
      B(1,7)=X1*Y1                                                     00001
      B(1,8)=X1*Y3                                                     00001
      B(1,9)=X3*Y3                                                     00001
      B(2,1)=X2*Y6                                                     00001
      B(2,2)=X2*Y5                                                     00001
      B(2,3)=X2*Y4                                                     00001
      B(2,4)=X1*Y5                                                     00001
      B(2,5)=X3*Y5                                                     00001
      B(2,6)=X3*Y4                                                     00001
      B(2,7)=X1*Y4                                                     00001
      B(2,8)=X1*Y6                                                     00001
      B(2,9)=X3*Y6                                                     00001
      B(3,1)=Y3*X5                                                     00001
      B(3,2)=Y2*X5                                                     00001
      B(3,3)=Y1*X5                                                     00001
      B(3,4)=Y2*X4                                                     00001
      B(3,5)=Y2*X6                                                     00001
      B(3,6)=Y1*X6                                                     00001
      B(3,7)=Y1*X4                                                     00001
      B(3,8)=Y3*X4                                                     00001
      B(3,9)=Y3*X6                                                     00001
      B(4,1)=X2*Y9                                                     00001
      B(4,2)=X2*Y8                                                     00001
      B(4,3)=X2*Y7                                                     00001
      B(4,4)=X1*Y8                                                     00001
      B(4,5)=X3*Y8                                                     00001
      B(4,6)=X3*Y7                                                     00001
      B(4,7)=X1*Y7                                                     00001
      B(4,8)=X1*Y9                                                     00001
      B(4,9)=X3*Y9                                                     00001
      B(5,1)=X5*Y6                                                     00001
      B(5,2)=X5*Y5                                                     00001
      B(5,3)=X5*Y4                                                     00001
```

```
      B(5,4)=X4*Y5                                                      00001
      B(5,5)=X6*Y5                                                      00001
      B(5,6)=X6*Y4                                                      00001
      B(5,7)=X4*Y4                                                      00001
      B(5,8)=X4*Y6                                                      00001
      B(5,9)=X6*Y6                                                      00001
      B(6,1)=Y3*X8
      B(6,2)=Y2*X8                                                      00001
      B(6,3)=Y1*X8
      B(6,4)=Y2*X7
      B(6,5)=Y2*X9
      B(6,6)=Y1*X9
      B(6,7)=Y1*X7                                                      00001
      B(6,8)=Y3*X7
      B(6,9)=Y3*X9                                                      00001
      GO TO 70                                                          00001
C     ELEMENT AREA AND THE MATRIX  B ARE AVAILABLE FOR THE ELEMENT M.
   70 WRITE(NX1)  B,AREA
      IF  (IEX .EQ. 1) GO TO 66
      CALL  ATBA  (PA2,B,CA2)
      WRITE  (NC2)  CA2
      GO TO  10
   66 CALL  IMCOEF  (NC2,IPA3T,XC,YC,W0,CONS1,CONS2,X0,Y0,NFLAT)
   10 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE INCDNC(NEL,NRCWS,NCOIS,NB,NT,NL,NR,NSCHM,ICTOUT)
      DIMENSION NPR(20),NODE(244,9),NN(20),MM(20),NODES(244),ITYPE(244)
      DIMENSION X(240),Y(240),IBC(244,3)
      COMMON /P15/ X,Y,NODE,ITYPE,IBC,NODES
      DO 32 I=1,NROWS
   32 NPR(I)=NCOLS
      IF(ICTOUT .NE. 1) GO TO 800
      NPC=NR-NL-1
      IL=NB+1
      IR=NT-1
      DO 33 I=IL,IR
   33 NPR(I)=NPR(I)-NPC
      NROW1=NROWS-1
      NN(1)=1
      DO 34 I=1,NROW1
   34 NN(I+1)=NN(I)+NPR(I)
      IR=NT-NB+1
      DO 35 I=1,IR
      II=NB+I-1
   35 MM(I)=NN(II)+NL-1
  800 NROW=1
      NSUM=NPR(1)
      IF( NSCHM .EQ. 1) GO TO 400
      DO 10 M=1,NEL
      IEL=ITYPE(M)
      IF( ICTOUT .NE. 1) GO TO 801
      IF( NROW .GE. NB .AND. NROW .LE. NT ) GO TO 100
  801 GO TO ( 1,2,3,4,5,6,7,8,9),IEL
    1 NODE(M,3)=NODES(M)
      NODE(M,2)=NODE(M,3)+1
      NODE(M,1)=NODE(M,3)+2
      NODE(M,7)=NODES(M)-NPR(NROW)
      NODE(M,4)=NODE(M,7)+1
      NODE(M,8)=NODE(M,7)+2
      NODE(M,6)=NODES(M)+NPR(NROW)
      NODE(M,5)=NODE(M,6)+1
      NODE(M,9)=NODE(M,6)+2
      GO TO 20
    2 NODE(M,4)=NODES(M)
      NODE(M,7)=NODE(M,4)-1
      NODE(M,8)=NODE(M,4)+1
      NODE(M,2)=NODES(M)+NPR(NROW)
      NODE(M,3)=NODE(M,2)-1
      NODE(M,1)=NODE(M,2)+1
      NODE(M,5)=NODE(M,2)+NPR(NROW)
      NODE(M,6)=NODE(M,5)-1
      NODE(M,9)=NODE(M,5)+1
      GO TO 20
    3 NODE(M,1)=NODES(M)
      NODE(M,2)=NODE(M,1)-1
      NODE(M,3)=NODE(M,1)-2
      NODE(M,8)=NODES(M)-NPR(NRCW)
      NODE(M,4)=NODE(M,8)-1
      NODE(M,7)=NODE(M,8)-2
```

```
      NODE(M,9)=NODES(M)+NPR(NROW)
      NODE(M,5)=NODE(M,9)-1
      NODE(M,6)=NODE(M,9)-2
      GO TO 20
    4 NODE(M,5)=NODES(M)
      NODE(M,6)=NODE(M,5)-1
      NODE(M,9)=NODE(M,5)+1
      NODE(M,2)=NODES(M)-NPR(NROW)
      NODE(M,3)=NODE(M,2)-1
      NODE(M,1)=NODE(M,2)+1
      NODE(M,4)=NODE(M,2)-NPR(NROW)
      NODE(M,7)=NODE(M,4)-1
      NODE(M,8)=NODE(M,4)+1
      GO TO 20
    5 NODE(M,2)=NODES(M)
      NODE(M,1)=NODE(M,2)+1
      NODE(M,3)=NODE(M,2)-1
      NODE(M,4)=NODES(M)-NPR(NROW)
      NODE(M,7)=NODE(M,4)-1
      NODE(M,8)=NODE(M,4)+1
      NODE(M,5)=NODE(M,2)+NPR(NROW)
      NODE(M,6)=NODE(M,5)-1
      NODE(M,9)=NODE(M,5)+1
      GO TO 20
    6 NODE(M,7)=NODES(M)
      NODE(M,4)=NODE(M,7)+1
      NODE(M,8)=NODE(M,7)+2
      NODE(M,3)=NODES(M)+NPR(NROW)
      NODE(M,2)=NODE(M,3)+1
      NODE(M,1)=NODE(M,3)+2
      NODE(M,6)=NODE(M,3)+NPR(NROW)
      NODE(M,5)=NODE(M,6)+1
      NODE(M,9)=NODE(M,6)+2
      GO TO 20
    7 NODE(M,8)=NODES(M)
      NODE(M,4)=NODE(M,8)-1
      NODE(M,7)=NODE(M,8)-2
      NODE(M,1)=NODE(M,8)+NPR(NROW)
      NODE(M,2)=NODE(M,1)-1
      NODE(M,3)=NODE(M,1)-2
      NODE(M,9)=NODE(M,1)+NPR(NROW)
      NODE(M,5)=NODE(M,9)-1
      NODE(M,6)=NODE(M,9)-2
      GO TO 20
    8 NODE(M,6)=NODES(M)
      NODE(M,5)=NODE(M,6)+1
      NODE(M,9)=NODE(M,6)+2
      NODE(M,3)=NODE(M,6)-NPR(NROW)
      NODE(M,2)=NODE(M,3)+1
      NODE(M,1)=NODE(M,3)+2
      NODE(M,7)=NODE(M,3)-NPR(NROW)
      NODE(M,4)=NODE(M,7)+1
      NODE(M,8)=NODE(M,7)+2
      GO TO 20
```

```
  9 NODE(M,9)=NODES(M)
    NODE(M,5)=NODE(M,9)-1
    NODE(M,6)=NODE(M,9)-2
    NODE(M,1)=NODE(M,9)-NPR(NROW)
    NODE(M,2)=NODE(M,1)-1
    NODE(M,3)=NODE(M,1)-2
    NODE(M,8)=NODE(M,1)-NPR(NROW)
    NODE(M,4)=NODE(M,8)-1
    NODE(M,7)=NODE(M,8)-2
    GO TO 20
100 I=NROW-NL+1
    IF( NODES(M) .LE. MM(I) ) GO TO 300
    GO TO ( 11,12,13,14,15,16,17,18,19),IEL
 11 NODE(M,3)=NODES(M)
    NODE(M,2)=NODE(M,3)+1
    NODE(M,1)=NODE(M,3)+2
    NODE(M,7)=NODE(M,3)-NPR(NROW)
    NODE(M,4)=NODE(M,7)+1
    NODE(M,8)=NODE(M,7)+2
    NODE(M,6)=NODE(M,3)+NPR(NROW+1)
    NODE(M,5)=NODE(M,6)+1
    NODE(M,9)=NODE(M,6)+2
    GO TO 20
 12 NODE(M,4)=NODES(M)
    NODE(M,7)=NODE(M,4)-1
    NODE(M,8)=NODE(M,4)+1
    NODE(M,2)=NODE(M,4)+NPR(NROW+1)
    NODE(M,3)=NODE(M,2)-1
    NODE(M,1)=NODE(M,2)+1
    NODE(M,5)=NODE(M,2)+NPR(NROW+2)
    NODE(M,6)=NODE(M,5)-1
    NODE(M,9)=NODE(M,5)+1
    GO TO 20
 13 NODE(M,1)=NODES(M)
    NODE(M,2)=NODE(M,1)-1
    NODE(M,3)=NODE(M,1)-2
    NODE(M,8)=NODE(M,1)-NPR(NROW)
    NODE(M,4)=NODE(M,8)-1
    NODE(M,7)=NODE(M,8)-2
    NODE(M,9)=NODE(M,1)+NPR(NROW+1)
    NODE(M,5)=NODE(M,9)-1
    NODE(M,6)=NODE(M,9)-2
    GO TO 20
 14 NODE(M,5)=NODES(M)
    NODE(M,6)=NODE(M,5)-1
    NODE(M,9)=NODE(M,5)+1
    NODE(M,2)=NODE(M,5)-NPR(NROW)
    NODE(M,3)=NODE(M,2)-1
    NODE(M,1)=NODE(M,2)+1
    NODE(M,4)=NODE(M,2)-NPR(NROW-1)
    NODE(M,7)=NODE(M,4)-1
    NODE(M,8)=NODE(M,4)+1
    GO TO 20
 15 NODE(M,2)=NODES(M)
```

```
      NODE(M,1)=NODE(M,2)+1
      NODE(M,3)=NODE(M,2)-1
      NODE(M,4)=NODE(M,2)-NPR(NROW)
      NODE(M,7)=NODE(M,4)-1
      NODE(M,8)=NODE(M,4)+1
      NODE(M,5)=NODE(M,2)+NPR(NROW+1)
      NODE(M,6)=NODE(M,5)-1
      NODE(M,9)=NODE(M,5)+1
      GO TO 20
   16 NODE(M,7)=NODES(M)
      NODE(M,4)=NODE(M,7)+1
      NODE(M,8)=NODE(M,7)+2
      NODE(M,3)=NODE(M,7)+NPR(NROW+1)
      NODE(M,2)=NODE(M,3)+1
      NODE(M,1)=NODE(M,3)+2
      NODE(M,6)=NODE(M,3)+NPR(NROW+2)
      NODE(M,5)=NODE(M,6)+1
      NODE(M,9)=NODE(M,6)+2
      GO TO 20
   17 NODE(M,8)=NODES(M)
      NODE(M,4)=NODE(M,8)-1
      NODE(M,7)=NODE(M,8)-2
      NODE(M,1)=NODE(M,8)+NPR(NROW+1)
      NODE(M,2)=NODE(M,1)-1
      NODE(M,3)=NODE(M,1)-2
      NODE(M,9)=NODE(M,1)  +NPR(NROW+2)
      NODE(M,5)=NODE(M,9)-1
      NODE(M,6)=NODE(M,9)-2
      GO TO 20
   18 NODE(M,6)=NODES(M)
      NODE(M,5)=NODE(M,6)+1
      NODE(M,9)=NODE(M,6)+2
      NODE(M,3)=NODE(M,6)-NPR(NROW)
      NODE(M,2)=NODE(M,3)+1
      NODE(M,1)=NODE(M,3)+2
      NODE(M,7)=NODE(M,3)-NPR(NROW-1)
      NODE(M,4)=NODE(M,7)+1
      NODE(M,8)=NODE(M,7)+2
      GO TO 20
   19 NODE(M,9)=NODES(M)
      NODE(M,5)=NODE(M,9)-1
      NODE(M,6)=NODE(M,9)-2
      NODE(M,1)=NODE(M,9)-NPR(NROW)
      NODE(M,2)=NODE(M,1)-1
      NODE(M,3)=NODE(M,1)-2
      NODE(M,8)=NODE(M,1)-NPR(NROW-1)
      NODE(M,4)=NODE(M,8)-1
      NODE(M,7)=NODE(M,8)-2
      GO TO 20
  300 GO TO( 21,22,23,24,25,26,27,28,29) ,IEL
   21 NODE(M,3)=NODES(M)
      NODE(M,2)=NODE(M,3)+1
      NODE(M,1)=NODE(M,3)+2
      NODE(M,7)=NODE(M,3)-NPR(NROW-1)
```

```
      NODE(M,4)=NODE(M,7)+1
      NODE(M,8)=NODE(M,7)+2
      NODE(M,6)=NODE(M,3)+NPR(NROW)
      NODE(M,5)=NODE(M,6)+1
      NODE(M,9)=NODE(M,6)+2
      GO TO 20
   22 NODE(M,4)=NODES(M)
      NODE(M,7)=NODE(M,4)-1
      NODE(M,8)=NODE(M,4)+1
      NODE(M,2)=NODE(M,4)+NPR(NROW+1)
      NODE(M,3)=NODE(M,2)-1
      NODE(M,1)=NODE(M,2)+1
      NODE(M,5)=NODE(M,2)+NPR(NROW+2)
      NODE(M,6)=NODE(M,5)-1
      NODE(M,9)=NODE(M,5)+1
      GO TO 20
   23 NODE(M,1)=NODES(M)
      NODE(M,2)=NODE(M,1)-1
      NODE(M,3)=NODE(M,1)-2
      NODE(M,8)=NODE(M,1)-NPR(NROW-1)
      NODE(M,4)=NODE(M,8)-1
      NODE(M,7)=NODE(M,8)-2
      NODE(M,9)=NODE(M,1)+NPR(NROW)
      NODE(M,5)=NODE(M,9)-1
      NODE(M,6)=NODE(M,9)-2
      GO TO 20
   24 NODE(M,5)=NODES(M)
      NODE(M,6)=NODE(M,5)-1
      NODE(M,9)=NODE(M,5)+1
      NODE(M,2)=NODE(M,5)-NPR(NROW-1)
      NODE(M,3)=NODE(M,2)-1
      NODE(M,1)=NODE(M,2)+1
      NODE(M,4)=NODE(M,2)-NPR(NROW-2)
      NODE(M,7)=NODE(M,4)-1
      NODE(M,8)=NODE(M,4)+1
      GO TO 20
   25 NODE(M,2)=NODES(M)
      NODE(M,1)=NODE(M,2)+1
      NODE(M,3)=NODE(M,2)-1
      NODE(M,4)=NODE(M,2)-NPR(NROW-1)
      NODE(M,7)=NODE(M,4)-1
      NODE(M,8)=NODE(M,4)+1
      NODE(M,5)=NODE(M,2)+NPR(NROW)
      NODE(M,6)=NODE(M,5)-1
      NODE(M,9)=NODE(M,5)+1
      GO TO 20
   26 NODE(M,7)=NODES(M)
      NODE(M,4)=NODE(M,7)+1
      NODE(M,8)=NODE(M,7)+2
      NODE(M,3)=NODE(M,7)+NPR(NROW)
      NODE(M,2)=NODE(M,3)+1
      NODE(M,1)=NODE(M,3)+2
      NODE(M,6)=NODE(M,3)+NPR(NROW+1)
      NODE(M,5)=NODE(M,6)+1
```

```
      NODE(M,9)=NODE(M,6)+2
      GO TO 20
   27 NODE(M,8)=NODES(M)
      NODE(M,4)=NODE(M,8)-1
      NODE(M,7)=NODE(M,8)-2
      NODE(M,1)=NODE(M,8)+NPR(NROW)
      NODE(M,2)=NODE(M,1)-1
      NODE(M,3)=NODE(M,1)-2
      NODE(M,9)=NODE(M,1)  +NPR(NROW+1)
      NODE(M,5)=NODE(M,9)-1
      NODE(M,6)=NODE(M,9)-2
      GO TO 20
   28 NODE(M,6)=NODES(M)
      NODE(M,5)=NODE(M,6)+1
      NODE(M,9)=NODE(M,6)+2
      NODE(M,3)=NODE(M,6)-NPR(NROW-1)
      NODE(M,2)=NODE(M,3)+1
      NODE(M,1)=NODE(M,3)+2
      NODE(M,7)=NODE(M,3)-NPR(NROW-2)
      NODE(M,4)=NODE(M,7)+1
      NODE(M,8)=NODE(M,7)+2
      GO TO 20
   29 NODE(M,9)=NODES(M)
      NODE(M,5)=NODE(M,9)-1
      NODE(M,6)=NODE(M,9)-2
      NODE(M,1)=NODE(M,9)-NPR(NROW-1)
      NODE(M,2)=NODE(M,1)-1
      NODE(M,3)=NODE(M,1)-2
      NODE(M,8)=NODE(M,1)-NPR(NROW-2)
      NODE(M,4)=NODE(M,8)-1
      NODE(M,7)=NODE(M,8)-2
      GO TO 20
   20 IF( NODES(M) .EQ. NSUM ) GO TO 200
      GO TO 10
  200 NROW=NROW+1
      IF(NROW .GT. NROWS) GO TO 10
      NSUM=NSUM+NPR(NROW)
   10 CONTINUE
      GO TO 90
  400 DO 60 M=1,NEL
      IEL=ITYPE(M)
      IF( ICTOUT .NE. 1) GO TO 802
      IF( NROW .GE. NB .AND. NROW .LE. NT) GO TO 500
  802 GO TO(61,62,63,64,65,66,67,68,69),IEL
   61 NODE(M,3)=NODES(M)
      NODE(M,7)=NODE(M,3)-1
      NODE(M,6)=NODE(M,3)+1
      NODE(M,2)=NODE(M,3)+NPR(NROW)
      NODE(M,4)=NODE(M,2)-1
      NODE(M,5)=NODE(M,2)+1
      NODE(M,1)=NODE(M,2)+NPR(NROW+1)
      NODE(M,8)=NODE(M,1)-1
      NODE(M,9)=NODE(M,1)+1
      GO TO 80
```

```
62 NODE(M,4)=NODES(M)
   NODE(M,2)=NODE(M,4)+1
   NODE(M,5)=NODE(M,4)+2
   NODE(M,7)=NODE(M,4)-NPR(NROW)
   NODE(M,3)=NODE(M,7)+1
   NODE(M,6)=NODE(M,7)+2
   NODE(M,8)=NODE(M,4)+NPR(NROW+1)
   NODE(M,1)=NODE(M,8)+1
   NODE(M,9)=NODE(M,8)+2
   GO TO 80
63 NODE(M,1)=NODES(M)
   NODE(M,8)=NODE(M,1)-1
   NODE(M,9)=NODE(M,1)+1
   NODE(M,2)=NODE(M,1)-NPR(NROW-1)
   NODE(M,4)=NODE(M,2)-1
   NODE(M,5)=NODE(M,2)+1
   NODE(M,3)=NODE(M,2)-NPR(NROW-2)
   NODE(M,7)=NODE(M,3)-1
   NODE(M,6)=NODE(M,3)+1
   GO TO 80
64 NODE(M,5)=NODES(M)
   NODE(M,2)=NODE(M,5)-1
   NODE(M,4)=NODE(M,5)-2
   NODE(M,9)=NODE(M,5)+NPR(NROW)
   NODE(M,1)=NODE(M,9)-1
   NODE(M,8)=NODE(M,9)-2
   NODE(M,6)=NODE(M,5)-NPR(NROW-1)
   NODE(M,3)=NODE(M,6)-1
   NODE(M,7)=NODE(M,6)-2
   GO TO 80
65 NODE(M,2)=NODES(M)
   NODE(M,4)=NODE(M,2)-1
   NODE(M,5)=NODE(M,2)+1
   NODE(M,1)=NODE(M,2)+NPR(NROW)
   NODE(M,8)=NODE(M,1)-1
   NODE(M,9)=NODE(M,1)+1
   NODE(M,3)=NODE(M,2)-NPR(NROW-1)
   NODE(M,7)=NODE(M,3)-1
   NODE(M,6)=NODE(M,3)+1
   GO TO 80
66 NODE(M,7)=NODES(M)
   NODE(M,3)=NODE(M,7)+1
   NODE(M,6)=NODE(M,7)+2
   NODE(M,4)=NODE(M,7)+NPR(NROW)
   NODE(M,2)=NODE(M,4)+1
   NODE(M,5)=NODE(M,4)+2
   NODE(M,8)=NODE(M,4)+NPR(NROW+1)
   NODE(M,1)=NODE(M,8)+1
   NODE(M,9)=NODE(M,8)+2
   GO TO 80
67 NODE(M,8)=NODES(M)
   NODE(M,1)=NODE(M,8)+1
   NODE(M,9)=NODE(M,8)+2
   NODE(M,4)=NODE(M,8)-NPR(NROW-1)
```

```
      NODE(M,2)=NODE(M,4)+1
      NODE(M,5)=NODE(M,4)+2
      NODE(M,7)=NODE(M,4)-NPR(NROW-2)
      NODE(M,3)=NODE(M,7)+1
      NODE(M,6)=NODE(M,7)+2
      GO TO 80
   68 NODE(M,6)=NODES(M)
      NODE(M,3)=NODE(M,6)-1
      NODE(M,7)=NODE(M,6)-2
      NODE(M,5)=NODE(M,6)+NPR(NROW)
      NODE(M,2)=NODE(M,5)-1
      NODE(M,4)=NODE(M,5)-2
      NODE(M,9)=NODE(M,5)+NPR(NROW+1)
      NODE(M,1)=NODE(M,9)-1
      NODE(M,8)=NODE(M,9)-2
      GO TO 80
   69 NODE(M,9)=NODES(M)
      NODE(M,1)=NODE(M,9)-1
      NODE(M,8)=NODE(M,9)-2
      NODE(M,5)=NODE(M,9)-NPR(NROW-1)
      NODE(M,2)=NODE(M,5)-1
      NODE(M,4)=NODE(M,5)-2
      NODE(M,6)=NODE(M,5)-NPR(NROW-2)
      NODE(M,3)=NODE(M,6)-1
      NODE(M,7)=NODE(M,6)-2
      GO TO 80
  500 I=NROW-NL+1
      IF( NODES(M) .LE. MM(I)) GO TO 600
      GO TO(71,72,73,74,75,76,77,78,79),IEL
   71 NODE(M,3)=NODES(M)
      NODE(M,7)=NODE(M,3)-1
      NODE(M,6)=NODE(M,3)+1
      NODE(M,2)=NODE(M,3)+NPR(NROW)
      NODE(M,4)=NODE(M,2)-1
      NODE(M,5)=NODE(M,2)+1
      NODE(M,1)=NODE(M,2)+NPR(NROW+1)
      NODE(M,8)=NODE(M,1)-1
      NODE(M,9)=NODE(M,1)+1
      GO TO 80
   72 NODE(M,4)=NODES(M)
      NODE(M,2)=NODE(M,4)+1
      NODE(M,5)=NODE(M,4)+2
      NODE(M,7)=NODE(M,4)-NPR(NROW)
      NODE(M,3)=NODE(M,7)+1
      NODE(M,6)=NODE(M,7)+2
      NODE(M,8)=NODE(M,4)+NPR(NROW+1)
      NODE(M,1)=NODE(M,8)+1
      NODE(M,9)=NODE(M,8)+2
      GO TO 80
   73 NODE(M,1)=NODES(M)
      NODE(M,8)=NODE(M,1)-1
      NODE(M,9)=NODE(M,1)+1
      NODE(M,2)=NODE(M,1)-NPR(NROW-1)
      NODE(M,4)=NODE(M,2)-1
```

```
      NODE(M,5)=NODE(M,2)+1
      NODE(M,3)=NODE(M,2)-NPR(NROW-2)
      NODE(M,7)=NODE(M,3)-1
      NODE(M,6)=NODE(M,3)+1
      GO TO 80
74    NODE(M,5)=NODES(M)
      NODE(M,2)=NODE(M,5)-1
      NODE(M,4)=NODE(M,5)-2
      NODE(M,6)=NODE(M,5)-NPR(NROW)
      NODE(M,3)=NODE(M,6)-1
      NODE(M,7)=NODE(M,6)-2
      NODE(M,9)=NODE(M,5)+NPR(NROW+1)
      NODE(M,1)=NODE(M,9)-1
      NODE(M,8)=NODE(M,9)-2
      GO TO 80
75    NODE(M,2)=NODES(M)
      NODE(M,4)=NODE(M,2)-1
      NODE(M,5)=NODE(M,2)+1
      NODE(M,3)=NODE(M,2)-NPR(NROW)
      NODE(M,7)=NODE(M,3)-1
      NODE(M,6)=NODE(M,3)+1
      NODE(M,1)=NODE(M,2)+NPR(NROW+1)
      NODE(M,8)=NODE(M,1)-1
      NODE(M,9)=NODE(M,1)+1
      GO TO 80
76    NODE(M,7)=NODES(M)
      NODE(M,3)=NODE(M,7)+1
      NODE(M,6)=NODE(M,7)+2
      NODE(M,4)=NODE(M,7)+NPR(NROW)
      NODE(M,2)=NODE(M,4)+1
      NODE(M,5)=NODE(M,4)+2
      NODE(M,8)=NODE(M,4)+NPR(NROW+1)
      NODE(M,1)=NODE(M,8)+1
      NODE(M,9)=NODE(M,8)+2
      GO TO 80
77    NODE(M,8)=NODES(M)
      NODE(M,1)=NODE(M,8)+1
      NODE(M,9)=NODE(M,8)+2
      NODE(M,4)=NODE(M,8)-NPR(NROW-1)
      NODE(M,2)=NODE(M,4)+1
      NODE(M,5)=NODE(M,4)+2
      NODE(M,7)=NODE(M,4)-NPR(NROW-2)
      NODE(M,3)=NODE(M,7)+1
      NODE(M,6)=NODE(M,7)+2
      GO TO 80
78    NODE(M,6)=NODES(M)
      NODE(M,3)=NODE(M,6)-1
      NODE(M,7)=NODE(M,6)-2
      NODE(M,5)=NODE(M,6)+NPR(NROW+1)
      NODE(M,2)=NODE(M,5)-1
      NODE(M,4)=NODE(M,5)-2
      NODE(M,9)=NODE(M,5)+NPR(NROW+2)
      NODE(M,1)=NODE(M,9)-1
      NODE(M,8)=NODE(M,9)-2
```

```
      GO TO 80
   79 NODE(M,9)=NODES(M)
      NODE(M,1)=NODE(M,9)-1
      NODE(M,8)=NODE(M,9)-2
      NODE(M,5)=NODE(M,9)-NPR(NROW)
      NODE(M,2)=NODE(M,5)-1
      NODE(M,4)=NODE(M,5)-2
      NODE(M,6)=NODE(M,5)-NPR(NROW-1)
      NODE(M,3)=NODE(M,6)-1
      NODE(M,7)=NODE(M,6)-2
      GO TO 80
  600 GO TO(81,82,83,84,85,86,87,88,89),IEL
   81 NODE(M,3)=NODES(M)
      NODE(M,7)=NODE(M,3)-1
      NODE(M,6)=NODE(M,3)+1
      NODE(M,2)=NODE(M,3)+NPR(NROW)
      NODE(M,4)=NODE(M,2)-1
      NODE(M,5)=NODE(M,2)+1
      NODE(M,1)=NODE(M,2)+NPR(NROW+1)
      NODE(M,8)=NODE(M,1)-1
      NODE(M,9)=NODE(M,1)+1
      GO TO 80
   82 NODE(M,4)=NODES(M)
      NODE(M,2)=NODE(M,4)+1
      NODE(M,5)=NODE(M,4)+2
      NODE(M,7)=NODE(M,4)-NPR(NROW-1)
      NODE(M,3)=NODE(M,7)+1
      NODE(M,6)=NODE(M,7)+2
      NODE(M,8)=NODE(M,4)+NPR(NROW)
      NODE(M,1)=NODE(M,8)+1
      NODE(M,9)=NODE(M,8)+2
      GO TO 80
   83 NODE(M,1)=NODES(M)
      NODE(M,8)=NODE(M,1)-1
      NODE(M,9)=NODE(M,1)+1
      NODE(M,2)=NODE(M,1)-NPR(NROW-1)
      NODE(M,4)=NODE(M,2)-1
      NODE(M,5)=NODE(M,2)+1
      NODE(M,3)=NODE(M,2)-NPR(NROW-2)
      NODE(M,7)=NODE(M,3)-1
      NODE(M,6)=NODE(M,3)+1
      GO TO 80
   84 NODE(M,5)=NODES(M)
      NODE(M,2)=NODE(M,5)-1
      NODE(M,4)=NODE(M,5)-2
      NODE(M,6)=NODE(M,5)-NPR(NROW-1)
      NODE(M,3)=NODE(M,6)-1
      NODE(M,7)=NODE(M,6)-2
      NODE(M,9)=NODE(M,5)+NPR(NROW)
      NODE(M,1)=NODE(M,9)-1
      NODE(M,8)=NODE(M,9)-2
      GO TO 80
   85 NODE(M,2)=NODES(M)
      NODE(M,4)=NODE(M,2)-1
```

```
      NODE(M,5)=NODE(M,2)+1
      NODE(M,3)=NODE(M,2)-NPR(NROW-1)
      NODE(M,7)=NODE(M,3)-1
      NODE(M,6)=NODE(M,3)+1
      NODE(M,1)=NODE(M,2)+NPR(NROW)
      NODE(M,8)=NODE(M,1)-1
      NODE(M,9)=NODE(M,1)+1
      GO TO 80
   86 NODE(M,7)=NODES(M)
      NODE(M,3)=NODE(M,7)+1
      NODE(M,6)=NODE(M,7)+2
      NODE(M,4)=NODE(M,7)+NPR(NROW)
      NODE(M,2)=NODE(M,4)+1
      NODE(M,5)=NODE(M,4)+2
      NODE(M,8)=NODE(M,4)+NPR(NROW+1)
      NODE(M,1)=NODE(M,8)+1
      NODE(M,9)=NODE(M,8)+2
      GO TO 80
   87 NODE(M,8)=NODES(M)
      NODE(M,1)=NODE(M,8)+1
      NODE(M,9)=NODE(M,8)+2
      NODE(M,4)=NODE(M,8)-NPR(NROW-1)
      NODE(M,2)=NODE(M,4)+1
      NODE(M,5)=NODE(M,4)+2
      NODE(M,7)=NODE(M,4)-NPR(NROW-2)
      NODE(M,3)=NODE(M,7)+1
      NODE(M,6)=NODE(M,7)+2
      GO TO 80
   88 NODE(M,6)=NODES(M)
      NODE(M,3)=NODE(M,6)-1
      NODE(M,7)=NODE(M,6)-2
      NODE(M,5)=NODE(M,6)+NPR(NROW)
      NODE(M,2)=NODE(M,5)-1
      NODE(M,4)=NODE(M,5)-2
      NODE(M,9)=NODE(M,5)+NPR(NROW+1)
      NODE(M,1)=NODE(M,9)-1
      NODE(M,8)=NODE(M,9)-2
      GO TO 80
   89 NODE(M,9)=NODES(M)
      NODE(M,1)=NODE(M,9)-1
      NODE(M,8)=NODE(M,9)-2
      NODE(M,5)=NODE(M,9)-NPR(NROW-1)
      NODE(M,2)=NODE(M,5)-1
      NODE(M,4)=NODE(M,5)-2
      NODE(M,6)=NODE(M,5)-NPR(NROW-2)
      NODE(M,3)=NODE(M,6)-1
      NODE(M,7)=NODE(M,6)-2
      GO TO 80
   80 IF(NODES(M) .EQ. NSUM) GO TO 700
      GO TO 60
  700 NROW=NROW+1
      IF( NROW .GT. NROWS) GO TO 60
      NSUM=NSUM+NPR(NROW)
   60 CONTINUE
```

```
90 CONTINUE
   RETURN
   END
```

```
      SUBROUTINE IMCOEF (NC2,IPA3T,XC,YC,W0,CONS1,CONS2,X0,Y0,NFLAT)
      DIMENSION PA2(12,12),PA3(12,12,12)
      DIMENSION PA2T(12,12),PA3T(12,12,12),B(6,9),CA2(27,27),ELN(6,6)
      DIMENSION QK(27,27),DQK(27,27),RE(27)
      COMMON /P11/ QK,DQK,RE,B,CA2
      COMMON /P13/ PA2,PA3
      COMMON /P14/ ELN,AREA,R
      COMMON /P16/ PA2T,PA3T
      DO 20 I=1,12
      DO 20 J=1,12
      PA2T(I,J)=PA2(I,J)
      DO 20 K=1,12
   20 PA3T(I,J,K)=PA3(I,J,K)
      WX=W0*CONS1*COS(CONS1*(XC-X0))*SIN(CONS2*(YC-Y0))
      WY=W0*CONS2*SIN(CONS1*(XC-X0))*COS(CONS2*(YC-Y0))
C
      PA2T(2,4)=-(ELN(1,2)*WY+ELN(1,3)*WX)/R*NFLAT+PA2(2,4)
      PA2T(2,8)=ELN(1,1)*WX+ELN(1,3)*WY+PA2(2,8)
      PA2T(2,9)=ELN(1,2)*WY+ELN(1,3)*WX+PA2(2,9)
      PA2T(3,4)=WX*(-ELN(3,3)/R+ELN(3,6)/(2.*R*R))*NFLAT+
     1WY*(-ELN(2,3)/R+ELN(2,6)/(2.*R*R))*NFLAT+PA2(3,4)
      PA2T(3,8)=WY*(ELN(3,3)-ELN(3,6)/(2.*R)*NFLAT)+
     1WX*(-ELN(1,6)/(2.*R)*NFLAT+ELN(1,3))+PA2(3,8)
      PA2T(3,9)=WX*(ELN(3,3)-ELN(3,6)/(2.*R)*NFLAT)+
     1WY*(-ELN(2,6)/(2.*R)*NFLAT+ELN(2,3))+PA2(3,9)
      PA2T(4,4)=(ELN(3,3)*WX*WX+2.*ELN(2,3)*WX*WY+ELN(2,2)*WY*WY)/(R*R)
     1*NFLAT+PA2(4,4)
      PA2T(4,5)=-WX*(ELN(3,3)/R+3.*ELN(3,6)/(2.*R*R))*NFLAT-
     1WY*(ELN(2,3)/R+3.*ELN(2,6)/(2.*R*R))*NFLAT+PA2(4,5)
      PA2T(4,6)=-WX*(ELN(3,5)/(R*R)+ELN(2,3)/R)*NFLAT-
     1WY*(ELN(2,2)/R+ELN(2,5)/(R*R))*NFLAT+PA2(4,6)
      PA2T(4,7)=-(ELN(2,2)*WY+ELN(2,3)*WX)/(R*R)*NFLAT+PA2(4,7)
      PA2T(4,8)=-(ELN(1,3)*WX*WX/R+(ELN(3,3)/R+ELN(1,2)/R)*WX*WY+
     1ELN(2,3)*WY*WY/R)*NFLAT+PA2(4,8)
      PA2T(4,9)=-(ELN(3,3)*WX*WX+2.*ELN(2,3)*WX*WY+ELN(2,2)*WY*WY)/R
     1*NFLAT+PA2(4,9)
      PA2T(4,10)=(ELN(3,4)*WX+ELN(2,4)*WY)/R*NFLAT+PA2(4,10)
      PA2T(4,11)=2.*(ELN(3,6)*WX+ELN(2,6)*WY)/R*NFLAT+PA2(4,11)
      PA2T(4,12)=(ELN(3,5)*WX+ELN(2,5)*WY)/R*NFLAT+PA2(4,12)
      PA2T(5,8)=WX*(ELN(1,3)+1.5*ELN(1,6)/R*NFLAT)+WY*(ELN(3,3)+1.5*
     1ELN(3,6)/R*NFLAT)+PA2(5,8)
      PA2T(5,9)=WX*(ELN(3,3)+1.5*ELN(3,6)/R*NFLAT)+WY*(ELN(2,3)+1.5*
     1ELN(2,6)/R*NFLAT)+PA2(5,9)
      PA2T(6,8)=WY*(ELN(2,3)+ELN(3,5)/R*NFLAT)+WX*(ELN(1,2)+
     1ELN(1,5)/R*NFLAT)+PA2(6,8)
      PA2T(6,9)=WX*(ELN(2,3)+ELN(3,5)/R*NFLAT)+WY*(    ELN(2,2)+
     1ELN(2,5)/R*NFLAT)+PA2(6,9)
      PA2T(7,8)=(WY*ELN(2,3)+WX*ELN(1,2))/R*NFLAT+PA2(7,8)
      PA2T(7,9)=(WY*    ELN(2,2)+WX*ELN(2,3))/R*NFLAT+PA2(7,9)
      PA2T(8,8)=WX*WX*ELN(1,1)+WY*WY*ELN(3,3)+WX*WY*ELN(1,3)*2.+PA2(8,8)
      PA2T(8,9)=WX*WX*ELN(1,3)+WY*WY*ELN(2,3)+WX*WY*(    ELN(3,3)+
     1ELN(1,2))+PA2(8,9)
      PA2T(8,10)=-WX*ELN(1,4)-WY*ELN(3,4)+PA2(8,10)
      PA2T(8,11)=-2.*(WX*ELN(1,6)+WY*ELN(3,6))+PA2(8,11)
```

```
      PA2T(8,12)=-WX*ELN(1,5)-WY*ELN(3,5)+PA2(8,12)
      PA2T(9,9)=WX*WX*ELN(3,3)+WY*WY*ELN(2,2)+WX*WY*ELN(2,3)*2.+PA2(9,9)
      PA2T(9,10)=-WX*ELN(3,4)-WY*ELN(2,4)+PA2(9,10)
      PA2T(9,11)=-2.*(WX*ELN(3,6)+WY*ELN(2,6))+PA2(9,11)
      PA2T(9,12)=-WX*ELN(3,5)-WY*ELN(2,5)+PA2(9,12)
      DO 10 I=1,11
      JL=I+1
      DO 10 J=JL,12
   10 PA2T(J,I)=PA2T(I,J)
      CALL ATBA (PA2T,B,CA2)
      WRITE (NC2) CA2
C
      PA3T(8,8,8)=ELN(1,1)*WX+ELN(1,3)*WY+PA3(8,8,8)
      PA3T(5,5,8)=WX*(ELN(1,1)-ELN(1,2))/12.+WY*(ELN(1,3)-ELN(2,3))/12.
     1+PA3(5,5,8)
      PA3T(3,5,8)=WX*(-ELN(1,1)+ELN(1,2))/12.+WY*(-ELN(1,3)+ELN(2,3))/
     112.+PA3(3,5,8)
      PA3T(3,3,8)=WX*(ELN(1,1)-ELN(1,2))/12.+WY*(ELN(1,3)-ELN(2,3))/12.
     1+PA3(3,3,8)
      PA3T(8,9,9)=WX*(ELN(1,2)+2.*ELN(3,3))/3.+WY*ELN(2,3)+PA3(8,9,9)
      PA3T(4,8,9)=WX*(-ELN(1,2)-2.*ELN(3,3))/(3.*R)*NFLAT-
     1ELN(2,3)*WY/R*NFLAT+PA3(4,8,9)
      PA3T(4,4,8)=WX*(ELN(1,2)+2.*ELN(3,3))/(3.*R*R)*NFLAT+
     1ELN(2,3)*WY/(R*R)*NFLAT+PA3(4,4,8)
      PA3T(4,8,8)=-WY*(ELN(1,2)+2.*ELN(3,3))/(3.*R)*NFLAT-
     1WX*ELN(1,3)/R*NFLAT+PA3(4,8,8)
      PA3T(4,5,5)=WX*(-ELN(1,3)+ELN(2,3))/(12.*R)*NFLAT+WY*(-ELN(1,2)+
     1ELN(2,2))/(12.*R)*NFLAT+PA3(4,5,5)
      PA3T(3,4,5)=WY*(ELN(1,2)-ELN(2,2))/(12.*R)*NFLAT+WX*(ELN(1,3)-
     1ELN(2,3))/(12.*R)*NFLAT+PA3(3,4,5)
      PA3T(3,3,4)=WY*(-ELN(1,2)+ELN(2,2))/(12.*R)*NFLAT+WX*(-ELN(1,3)+
     1ELN(2,3))/(12.*R)*NFLAT+PA3(3,3,4)
      PA3T(8,8,9)=WY*(ELN(1,2)+2.*ELN(3,3))/3.+WX*ELN(1,3)+PA3(8,8,9)
      PA3T(5,5,9)=WY*(ELN(1,2)-ELN(2,2))/12.+WX*(ELN(1,3)-ELN(2,3))/12.
     1+PA3(5,5,9)
      PA3T(3,5,9)=WY*(-ELN(1,2)+ELN(2,2))/12.+WX*(-ELN(1,3)+
     1ELN(2,3))/12.+PA3(3,5,9)
      PA3T(3,3,9)=WY*(ELN(1,2)-ELN(2,2))/12.+WX*(ELN(1,3)-
     1ELN(2,3))/12.+PA3(3,3,9)
      PA3T(4,9,9)=-ELN(2,2)*WY/R*NFLAT-ELN(2,3)*WX/R*NFLAT+PA3(4,9,9)
      PA3T(4,4,9)=WX*ELN(2,3)/(R*R)*NFLAT+WY*ELN(2,2)/(R*R)*NFLAT
     1+PA3(4,4,9)
      PA3T(4,4,4)=(-ELN(2,2)*WY-ELN(2,3)*WX)/(R**3)*NFLAT+PA3(4,4,4)
      PA3T(9,9,9)=ELN(2,2)*WY+ELN(2,3)*WX+PA3(9,9,9)
      DO 30 I=1,12
      DO 30 J=1,12
      DO 30 K=1,12
      PA3T(J,I,K)=PA3T(I,J,K)
      PA3T(I,K,J)=PA3T(I,J,K)
   30 CONTINUE
      WRITE (IPA3T) PA3T
      RETURN
      END
      SUBROUTINE MULTOC(A,B,C,D,Q,MAXA,NAV,NEQB,NAR,LBAND,NBLOCK,
```

```
     1MI,NSTIF,KSTIF,NC,KEX)
      DIMENSION A(NAV),B(NAV),C(NEQB),D(NAV),Q(NAR),MAXA(MI)
C     THIS PROGRAM MULTIPLIES (A)*(X)=(C). THE VECTOR (X) IS
C     ASSUMED TO BE CONTAINED IN MEMORY. THE VECTOR (C) AND THE
C     SQUARE MATRIX (A) ARE ASSUMED TO BE STORED ON A TAPE IN BLOCKS.
C     THIS PROGRAM IS RESTRICTED TO VALUES OF NEQB GRATER THAN OR
C     EQUAL TO LBAND.
      DO 73 I=1,NEQB
   73 C(I)=0.
      LB1=LBAND+1
      REWIND NSTIF
      REWIND KSTIF
      REWIND NC
      READ (NSTIF) B
      IF (KEX .NE. 1) GO TO 50
      READ (KSTIF) D
      GO TO 51
   50 READ (KSTIF) D,MAXA
C     PROCESS THE FIRS BLOCK (NEQB. GE. LBAND)
   51 N=1
      DO 70 I=1,LBAND
      II=I
      LL=II-NEQB*(N-1)
      NN=NEQB*LBAND+LL
      DO 71 J=1,I
        II=J
        JJ=I-J+1
        KK=II-NEQB*(N-1)
        MM=KK+NEQB*(JJ-1)
   71 C(LL)=C(LL)+B(MM)*Q(J)
      JL1=I+1
      JR=I+LBAND-1
      DO 72 J=JL1,JR
        II=I
        JJ=J-I+1
        KK=II-NEQB*(N-1)
        MM=KK+NEQB*(JJ-1)
   72 C(LL)=C(LL)+B(MM)*Q(J)
   70 D(NN)=B(NN)-C(LL)
      IF (NEQB .EQ. LBAND) GO TO 100
      DO 80 I=LB1,NEQB
       II=I
       LL=II-NEQB*(N-1)
       NN=NEQB*LBAND+LL
       JL=I-LBAND+1
      DO 81 J=JL,I
       II=J
       JJ=I-J+1
       KK=II-NEQB*(N-1)
       MM=KK+NEQB*(JJ-1)
   81 C(LL)=C(LL)+B(MM)*Q(J)
      JL1=I+1
      JR=I+LBAND-1
      DO 82 J=JL1,JR
```

```
      II=I
      JJ=J-I+1
      KK=II-NEQB*(N-1)
      MM=KK+NEQB*(JJ-1)
   82 C(LL)=C(LL)+B(MM)*Q(J)
   80 D(NN)=B(NN)-C(LL)
  100 IF (KEX .NE. 1) GO TO 55
      WRITE (NC) D
      GO TO 56
   55 WRITE (NC) D,MAXA
   56 DO 60 N=2,NBLOCK
      DO 61 I=1,NEQB
   61 C(I)=0.
      IR=NEQB*LB1
      DO 69 I=1,IR
   69 A(I)=B(I)
      READ (NSTIF) B
      IF (KEX .NE. 1) GO TO 52
      READ (KSTIF) D
      GO TO 53
   52 READ (KSTIF) D,MAXA
   53 IL=NEQB*(N-1)+1
      IR=NEQB*(N-1)+LBAND-1
      DO 62 I=IL,IR
        II=I
        LL=II-NEQB*(N-1)
        NN=NEQB*LBAND+LL
       JL=I-LBAND+1
       JR=NEQB*(N-1)
      DO 63 J=JL,JR
       II=J
       JJ=I-J+1
       KK=II-NEQB*(N-2)
       MM=KK+NEQB*(JJ-1)
   63 C(LL)=C(LL)+A(MM)*Q(J)
      JL=JR+1
      JR=I
      DO 64 J=JL,JR
       II=J
       JJ=I-J+1
       KK=II-NEQB*(N-1)
       MM=KK+NEQB*(JJ-1)
   64 C(LL)=C(LL)+B(MM)*Q(J)
       JL=JR+1
       JR=I+LBAND-1
      DO 65 J=JL,JR
        II=I
        JJ=J-I+1
        KK=II-NEQB*(N-1)
        MM=KK+NEQB*(JJ-1)
   65 C(LL)=C(LL)+B(MM)*Q(J)
   62 D(NN)=B(NN)-C(LL)
C     THIS COMPLETES THE FIRST ROW-SUM N .GT. 1
      IL=NEQB*(N-1)+LBAND
```

AD-A071 649   DAYTON UNIV OHIO   F/G 20/11

COLLAPSE LOAD ANALYSIS FOR PLATES AND SHELLS.(U)

MAY 79   N R BAULD, K SATYAMURTHY   F33615-76-C-3145

UNCLASSIFIED   AFFDL-TR-79-3038   NL

3 OF 3

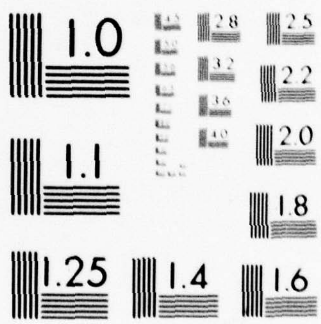
AD A071649

END

DATE FILMED

8-79

DDC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

```
      IR=NEQB*N
      DO 66 I=IL,IR
         II=I
         LL=II-NEQB*(N-1)
         NN=NEQB*LBAND+LL
         JL=I-LBAND+1
         JR=I
      DO 67 J=JL,JR
        II=J
        JJ=I-J+1
        KK=II-NEQB*(N-1)
        MM=KK+NEQB*(JJ-1)
   67 C(LL)=C(LL)+B(MM)*Q(J)
         JL=JR+1
         JR=I+LBAND-1
      DO 68 J=JL,JR
         II=I
         JJ=J-I+1
         KK=II-NEQB*(N-1)
         MM=KK+NEQB*(JJ-1)
   68 C(LL)=C(LL)+B(MM)*Q(J)
   66 D(NN)=B(NN)-C(LL)
C     THIS COMPLETES THE N TH BLOCK.
      IF (KEX .NE. 1) GO TO 54
      WRITE (NC) D
      GO TO 60
   54 WRITE (NC) D,MAXA
   60 CONTINUE
      RETURN
      END
      SUBROUTINE BCS(A,B,MAXA,NAV,NEQB,LBAND,MI,KEX,NBLOCK,NC,ND,MAXB)
      DIMENSION A(NAV),B(NAV),ID(400),MAXA(MI),MAXB(MI),NBC(9)
      COMMON /P17/ ID,IK,NBC
      LB1=LBAND+1
      N=1
      IF (KEX .NE. 1) GO TO 50
      READ (NC) B
      GO TO 51
   50 READ (NC) B,MAXB
   51 DO 10 J=1,IK
         IF(ID(J) .GT. LBAND .OR. ID(J) .LE. 0) GO TO 10
         II=ID(J)
         KK=II-NEQB*(N-1)
         B(KK)=1.
      DO 11 JJ=2,LBAND
         MM=KK+NEQB*(JJ-1)
         B(MM)=0.
         NN=II+1-JJ
         IF (NN .LE. 0) GO TO 12
         NN=NN+(JJ-1)*NEQB
         B(NN)=0.
   12 MM=KK+NEQB*LBAND
         B(MM)=0.
   11 CONTINUE
```

```
 10 CONTINUE
    IF (NEQB .EQ. LBAND) GO TO 100
    DO 20 J=1,IK
       IF (ID(J) .GT. NEQB .OR. ID(J) .LE. LBAND) GO TO 20
       II=ID(J)
       KK=II-NEQB*(N-1)
       B(KK)=1.
    DO 21 JJ=2,LBAND
       MM=KK+NEQB*(JJ-1)
       NN=II+(JJ-1)*(NEQB-1)
       B(MM)=0.
 21 B(NN)=0.
       MM=KK+NEQB*LBAND
       B(MM)=0.
 20 CONTINUE
100 DO 60 N=2,NBLOCK
       DO 69 I=1,NAV
 69     A(I)=B(I)
    IF (KEX .NE. 1) GO TO 52
     READ (NC) B
    GO TO 53
 52 DO 80 I=1,MI
 80 MAXA(I)=MAXB(I)
    READ (NC) B,MAXB
 53 DO 1 K=1,IK
       IF (ID(K) .GT. NEQB*N .OR. ID(K) .LE. NEQB*(N-1)) GO TO 1
       KK=ID(K)-NEQB*(N-1)
       IF (KK .GE. LBAND) GO TO 2
       KK1=KK+1
    DO 4 JJ=1,KK
    I=ID(K)+1-JJ
    II=I-NEQB*(N-1)
    MM=II+NEQB*(JJ-1)
  4 B(MM)=0.
    DO 5 JJ=KK1,LBAND
    I=ID(K)+1-JJ
    II=I-NEQB*(N-2)
    MM=II+NEQB*(JJ-1)
  5 A(MM)=0.
    DO 3 JJ=2,LB1
    MM=KK+NEQB*(JJ-1)
  3 B(MM)=0.
       B(KK)=1.
    GO TO 1
  2 DO 6 JJ=2,LBAND
       MM=KK+NEQB*(JJ-1)
       NN=ID(K)+(JJ-1)*(NEQB-1)-NEQB*(N-1)
       B(MM)=0.
       B(NN)=0.
       MM=KK+NEQB*LBAND
  6    B(MM)=0.
    B(KK)=1.
  1 CONTINUE
    IF (KEX .NE. 1) GO TO 54
```

```
      WRITE (ND) A
      GO TO 60
   54 WRITE (ND) A,MAXA
   60 CONTINUE
      IF (KEX .NE. 1) GO TO 55
      WRITE (ND) B
      GO TO 56
   55 WRITE (ND) B,MAXB
   56 CONTINUE
      RETURN
      END
      SUBROUTINE ATBA (A,B,C)
      DIMENSION A(12,12),B(6,9),BB(12,27),C(27,27)
      DO 10 NN=1,3
      NEVEN=NN-1
      JR=3
      IF (NN .EQ. 1) JR=6
      JK=6-6*NEVEN
      IF (NN .EQ. 3) JK=3
      DO 1 K=1,9
      KK=K+NEVEN
      DO 2 I=1,12
      BB(I,KK)=0.0
      DO 2 J=1,JR
      JJ=J+JK
    2 BB(I,KK)=BB(I,KK)+A(I,JJ)*B(J,K)
    1 NEVEN=NEVEN+2
   10 CONTINUE
      DO 20 NN=1,3
      NEVEN=NN-1
      JR=3
      IF (NN .EQ. 1) JR=6
      JK=6-6*NEVEN
      IF (NN .EQ. 3) JK=3
      DO 4 I=1,9
      KK=I+NEVEN
      DO 3 K=KK,27
      C(KK,K)=0.0
      DO 3 J=1,JR
      JJ=J+JK
    3 C(KK,K)=C(KK,K)+B(J,I)*BB(JJ,K)
    4 NEVEN=NEVEN+2
   20 CONTINUE
      DO 30 I=1,26
      IL=I+1
      DO 30 J=IL,27
   30 C(J,I)=C(I,J)
      RETURN
      END
      SUBROUTINE SMALL(Q,CC,SC,NEQB,NBLOCK,LEVEL,ITER,BIG,NL,NDIS,NAR,
     1NUMNP)
      DIMENSION Q(NAR),CC(NEQB),SC(NBLOCK)
      DO 7040 K=1,NBLOCK
      READ (NL) (CC(I),I=1,NEQB)
```

```
      NBIG=1
      DO 7041 I=1,NEQB
      IF ((ABS(CC(I))) .GT. (ABS(CC(NBIG)))) NBIG=I
      II=I+(NBLOCK-K)*NEQB
 7041 Q(II)=Q(II)+CC(I)
      SC(K)=CC(NBIG)
 7040 CONTINUE
C
      NBIG=1
      DO 7042 K=1,NBLOCK
      IF(ABS(SC(K)) .GT. ABS(SC(NBIG))) NBIG=K
      IL=(K-1)*NEQB+1
      IR=K*NEQB
 7042 WRITE(NDIS) (Q(I),I=IL,IR)
      BIG=SC(NBIG)
      WRITE (3,7052) LEVEL,ITER
 7052 FORMAT (1H1,2X,'LOAD LEVEL=',I5,5X,'ITERATION NUMBER=',I5)
      WRITE (3,7053)
 7053 FORMAT (1H0,2X,'NODAL POINT NO.',4X,'W-DISPLACEMENT',4X,
     1'U-DISPLACEMENT',4X,'V-DISPLACEMENT')
      DO 7050 K=1,NUMNP
      WRITE (3,7051) K,Q(3*K-2),Q(3*K-1),Q(3*K)
 7051 FORMAT (1H0,2X,I8,9X,E16.6,2X,E16.6,2X,E16.6)
 7050 CONTINUE
      RETURN
      END
      SUBROUTINE IBAND(NEL,LBAND)
      DIMENSION NODE(244,9),ITYPE(244),X(240),Y(240),IBC(244,3)
      DIMENSION NODES(244)
      COMMON /P15/ X,Y,NODE,ITYPE,IBC,NODES
      LARGE=1
      DO 11 M=1,NEL
    2 LBIG=IABS(NODE(M,7)-NODE(M,9))
   10 IF(LBIG .GT. LARGE) LARGE=LBIG
   11 CONTINUE
      LBAND=3*(LARGE+1)
      RETURN
      END
       SUBROUTINE SESOL (A,B,MAXA,NEQ,MA,NV,NBLOCK,NEQB,NAV,MI,NSTIF,
     1                   NRED,NL,NR,KBLOCK,NN,KEX,MAXB,DET)
C
       DIMENSION A(NAV),B(NAV),MAXA(MI),MAXB(MI)
C
      DET=1.0
      MM=1
       MA2=MA - 2
      IF(MA2.EQ.0) MA2=1
       INC=NEQB - 1
       NWA=NEQB*MA
       NTB=(MA-2)/NEQB + 1
       NEB=NTB*NEQB
       NEBT=NEB + NEQB
       NWV=NEQB*NV
       NWVV=NEBT*NV
```

```
C
      N1=NL
      N2=NR
      REWIND NSTIF
      REWIND NRED
      REWIND N1
      REWIND N2
C
      GO TO ( 710,721),KEX
C       MAIN LOOP OVER ALL BLOCKS
  710 DO 600 NJ=1,NBLOCK
        IF (NJ.NE.1) GO TO 10
      READ (NSTIF) A
      IF(NEQ.GT.1) GO TO 100
      MAXA(1)=1
      WRITE(NRED) A,MAXA
      IF(A(1)) 1,174,3
    1 KK=1
      WRITE (3,1010) KK,A(1)
    3 DO 5 L=1,NV
    5 A(1+L)=A(1+L)/A(1)
      KR=1+NV
      WRITE(NL) (A(KK),KK=2,KR)
      RETURN
   10   IF (NTB.EQ.1) GO TO 100
        REWIND N1
        REWIND N2
        READ (N1) A
C
C       FIND COLUMN HEIGHTS
  100   KU=1
        KM=MINO(MA,NEQB)
        MAXA(1)=1
        DO 110 N=2,MI
        IF (N-MA) 120,120,130
  120   KU=KU + NEQB
        KK=KU
      MM=MINO(N,KM)
        GO TO 140
  130   KU=KU + 1
        KK=KU
        IF (N-NEQB) 140, 140, 136
  136   MM=MM - 1
  140   DO 160 K=1,MM
        IF (A(KK)) 110,160,110
  160   KK=KK - INC
  110   MAXA(N)=KK
C
        IF (A(1)) 172,174,176
  174   KK=(NJ-1)*NEQB + 1
        IF (KK.GT.NEQ) GO TO 590
      WRITE (3,1000) KK
        STOP
  172   KK=(NJ-1)*NEQB + 1
```

```
      WRITE (3,1010) KK,A(1)
C
C      FACTORIZE LEADING BLOCK
 176   DO 200 N=2,NEQB
       NH=MAXA(N)
       IF (NH-N) 200,200,210
 210   KL=N + INC
       K=N
       D=0.
       DO 220 KK=KL,NH,INC
       K=K - 1
       C=A(KK)/A(K)
       D=D + C*A(KK)
 220   A(KK)=C
       A(N)=A(N) - D
C
       IF (A(N)) 222,224,230
 224   KK=(NJ-1)*NEQB + N
       IF (KK.GT.NEQ) GO TO 590
      WRITE (3,1000) KK
       STOP
 222   KK=(NJ-1)*NEQB + N
      WRITE (3,1010) KK,A(N)
C
 230   IC=NEQB
       DO 240 J=1,MA2
       MJ=MAXA(N+J) - IC
       IF (MJ-N) 240,240,280
 280   KU=MINO(MJ,NH)
       KN=N + IC
       C=0.
       DO 300 KK=KL,KU,INC
 300   C=C + A(KK)*A(KK+IC)
       A(KN)=A(KN) - C
 240   IC=IC + NEQB
C
C
 200   CONTINUE
C
C      CARRY OVER INTO TRAILING BLOCKS
       DO 400 NK=1,NTB
       IF ((NK+NJ).GT.NBLOCK) GO TO 400
       NI=N1
       IF ((NJ.EQ.1).OR.(NK.EQ.NTB)) NI=NSTIF
      READ (NI) B
       ML=NK*NEQB + 1
       MR=MINO((NK+1)*NEQB,MI)
      IF(MA.EQ.1) ML=MR
       MD=MI - ML
       KL=NEQB + (NK-1)*NEQB*NEQB
       N=1
C
       DO 500 M=ML,MR
       NH=MAXA(M)
```

```
      KL=KL + NEQB
      IF (NH-KL) 505,510,510
510   K=NEQB
      D=0.
      DO 520 KK=KL,NH,INC
      C=A(KK)/A(K)
      D=D + C*A(KK)
      A(KK)=C
520   K=K - 1
      B(N)=B(N) - D
     IF (MD) 505,505,530
530   IC=NEQB
      DO 540 J=1,MD
      MJ=MAXA(M+J) - IC
      IF (MJ-KL) 540,550,550
550   KU=MINO(MJ,NH)
      KN=N + IC
      C=0.
      DO 575 KK=KL,KU,INC
575   C=C + A(KK)*A(KK+IC)
      B(KN)=B(KN) - C
540   IC=IC + NEQB
C
C
505   MD=MD - 1
500   N=N + 1
C
      IF (NTB.NE.1) GO TO 560
      WRITE (NRED) A,MAXA
     NN=NEQB
     IF (NJ .EQ. KBLOCK) NN=NN-(KBLOCK-1)*NEQB
     DO 5001 II=1,NN
5001 DET=DET*A(II)/ ABS(A(II))
      DO 570 I=1,NAV
570   A(I)=B(I)
      GO TO 600
560   WRITE (N2) B
C
400   CONTINUE
C
      M=N1
      N1=N2
      N2=M
590   WRITE (NRED) A,MAXA
C
     NN=NEQB
     IF (NJ .EQ. KBLOCK) NN=NN-(KBLOCK-1)*NEQB
     DO 5000 II=1,NN
5000 DET=DET*A(II)/ ABS(A(II))
600   CONTINUE
     DET=DET* ABS(A(NN))
     WRITE (3,5111) DET
5111 FORMAT (1H0,4X,'DETERMINANT OF THE COEFFICIENT MATRIX',F16.6)
C
```

```
      IF( KEX .EQ. 1 ) GO TC 720
  721 M=N1
      N1=N2
      N2=M
  720 REWINC NSTIF
      REWIND NRED
      REWIND N1
      REWIND N2
      DO 60 NJ=1,NBLOCK
      IF(NJ .NE. 1) GO TO 61
      READ (NRED) A,MAXA
   61 IF(NTB .EQ. 1) GO TC 62
      REWINC N1
      REWIND N2
      READ(N1) A
   62 DO 63 N=2,NEQB
      NH=MAXA(N)
      IF((NH-N) .LE. 0) GO TO 63
      KL=N+INC
      K=N+NWA
      DO 64 L=1,NV
      KJ=K
      C=0.
      DO 65 KK=KL,NH,INC
      KJ=KJ-1
   65 C=C+A(KK)*A(KJ)
      A(K)=A(K)-C
   64 K=K+NEQB
   63 CONTINUE
      DO 66 NK=1,NTB
      IF((NK+NJ) .GT. NBLOCK) GO TO 66
      NI=N1
      IF((NJ .EQ. 1) .OR. (NK .EQ. NTB)) NI=NRED
      READ (NI) B,MAXB
      ML=NK*NEQB+1
      MR=MINO((NK+1)*NECB,MI)
      IF(MA .EQ. 1) ML=MR
      KL=NEQB+(NK-1)*NECB*NECB
      N=1
      DO 67 M=ML,MR
      NH=MAXA(M)
      KL=KL+NEQB
      IF((NH-KL) .LT. 0) GO TO 67
      KN=N+NWA
      K=NEQB+NWA
      DO 68 L=1,NV
      KJ=K
      C=0.
      DO 69 KK=KL,NH,INC
      C=C+A(KK)*A(KJ)
   69 KJ=KJ-1
      B(KN)=B(KN)-C
      KN=KN+NEQB
   68 K=K+NEQB
```

```
   67 N=N+1
      IF(NTE .NE. 1) GO TO 70
      WRITE (NSTIF) A,MAXA
      DO 71 I=1,NAV
   71 A(I)=B(I)
      DO 999 I=1,MI
  999 MAXA(I)=MAXB(I)
      GO TO 60
   70 WRITE(N2) B
   66 CONTINUE
      M=N1
      N1=N2
      N2=M
      WRITE (NSTIF) A,MAXA
   60 CONTINUE
C       VECTOR BACKSUBSTITUTION
        DO 700 K=1,NWVV
 700    B(K)=0.
        REWIND NL
C
        DO 800 NJ=1,NBLOCK
      BACKSPACE NSTIF
      READ (NSTIF) A,MAXA
      BACKSPACE NSTIF
        K=NEBT
        DO 810 L=1,NV
        DO 820 I=1,NEB
        B(K)=B(K-NEQB)
 820    K=K - 1
 810    K=K + NEBT + NEB
        KN=0
        KK=NWA
        NDIF=NEQB
        IF  (NJ.EQ.1) NDIF=NEQB - (NBLOCK*NEQB - NEQ)
        DO 855 L=1,NV
        DO 850 K=1,NDIF
 850    B(KN+K)=A(KK+K)/A(K)
        KK=KK + NEQB
 855    KN=KN + NEBT
      IF(MA.EQ.1) GO TO 915
        ML=NEQB + 1
        KL=NEQB
        DO 860 M=ML,MI
        KL=KL + NEQB
        KU=MAXA(M)
        IF  (KU-KL) 860,870,870
 870    K=NEQB
        KM=M
        DO 880 L=1,NV
        KJ=K
        DO 890 KK=KL,KU,INC
        B(KJ)=B(KJ) - A(KK)*B(KM)
 890    KJ=KJ - 1
        KM=KM + NEBT
```

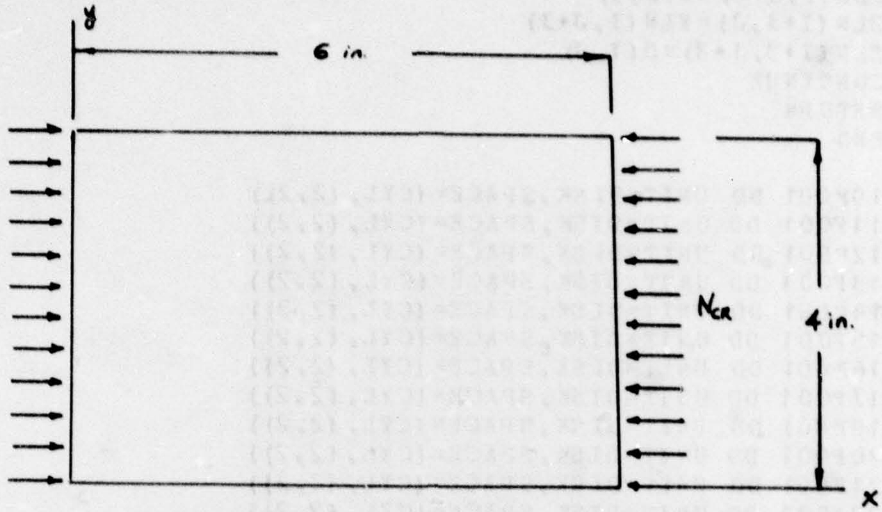
```
880    K=K + NEBT
860    CONTINUE
       N=NEQB
       DO 910 I=2,NEQB
       KL=N + INC
       KU=MAXA(N)
       IF (KU-KL) 910,920,920
920    K=N
       DO 930 L=1,NV
       KJ=K
       DO 940 KK=KL,KU,INC
       KJ=KJ - 1
940    B(KJ)=B(KJ) - A(KK)*B(K)
930    K=K + NEBT
910    N=N - 1
C
  915 KK=0
       KN=0
       DO 950 L=1,NV
       DO 960 K=1,NEQB
       KK=KK + 1
960    A(KK)=B(KN+K)
950    KN=KN + NEBT
C
       WRITE (NL) (A(K),K=1,NWV)
800    CONTINUE
C
 1000 FORMAT (// 46H    STOP  ***  ZERO DIAGONAL ENCOUNTERED DURING,
     1             18H EQUATION SOLUTION, /
     2         13X,18H EQUATION NUMBER =, I6 )
 1010 FORMAT (/  50H WARNING ***  NEGATIVE DIAGONAL ENCOUNTERED DURING,
     1             18H EQUATION SOLUTION, /
     2         13X,18H EQUATION NUMBER =, I6, 5X, 7HVALUE =, E20.8 )
C
       RETURN
       END
      SUBROUTINE BCID(NEL,NEQ,NBLOCK,NEQB,KBLOCK,NN)
      DIMENSION NODE(244,9),ITYPE(244),X(240),Y(240),IBC(244,3)
      DIMENSION ID(400),NBC(9),NODES(244)
      COMMON /P15/ X,Y,NODE,ITYPE,IBC,NODES
      COMMON /P17/ ID,IK,NBC
      IK=0
      NBC(1)=3
      NBC(2)=4
      NBC(3)=1
      NBC(4)=5
      NBC(5)=2
      NBC(6)=7
      NBC(7)=8
      NBC(8)=6
      NBC(9)=9
      DO 120 M=1,NEL
      IEL=ITYPE(M)
      NP=NBC(IEL)
```

```
      DO 121 I=1,3
      IF (IBC(M,I) .EQ. 0) GO TO 121
      IK=IK+1
      ID(IK)=3*NODE(M,NE)+I-3
  121 CONTINUE
  120 CONTINUE
      IIK=IK
      NN=NEQ+1
      IIK=IIK+1
    2 NN=NN-1
      IIK=IIK-1
      IF (ID(IIK) .EQ. NN) GO TO 2
      DO 1 NJ=1,NBLOCK
    1 IF (NN .GT. (NJ-1)*NEQB .AND. NN .LE. NJ*NEQB) KBLOCK=NJ
      RETURN
      END
      SUBROUTINE ELPROP
      DIMENSION ELN(6,6)
      DIMENSION Q(10,3,3),D(3,3),A(3,3),B(3,3),H(10)
      COMMON /P14/ ELN,AREA,R
C
C     KN=TOTAL NUMBER OF LAYERS.
C     H(1),H(2),H(3),...... ARE THE DISTANCES OF THE LAYERS FROM
C     THE REFERENCE PLANE.
      READ (1,10) KN
   10 FORMAT (I5)
      KL=KN+1
      READ (1,13) (H(I),I=1,KL)
   13 FORMAT (F10.0)
      WRITE (3,14) KN
   14 FORMAT (1H1,2X,'TOTAL NUMBER OF LAYERS',I5)
      WRITE (3,11)
   11 FORMAT (1H0,5X,'LAYER DISTANCES FROM THE REFERENCE SURFACE')
      WRITE (3,12)
   12 FORMAT (1H0,2X,'POSITIVE FOR LAYER SURFACES TOWARD THE CENTER
     1OF CURVATURE')
      WRITE (3,16) (H(I),I=1,KL)
   16 FORMAT (5X,F16.6)
C     E1 IS THE MODULUS OF ELASTICITY IN THE FIBER DIRECTION.
C     E2 IS THE MODULUS OF ELASTICITY IN THE DIRECTION PERPENDICULAR
C     TO THE FIBER DIRECTION.
C     ANU1 IS THE POISSON'S RATIO IN THE 12 DIRECTION.
C     ANU2 IS THE POISSON'S RATIO IN THE 21 DIRECTION.
C     G IS THE SHEAR MODULUS.
C     TT IS THE ORIENTATION OF THE FIBER WITH RESPECT TO X-DIRECTION.
      WRITE (3,17)
   17 FORMAT (1H0,5X,'E1',15X,'E2',16X,'NU(1,2)',9X,'NU(2,1)',4X,
     1'G(1,2)',9X,'THETA')
      DO 101 N=2,KL
      READ (1,20) E1,E2,ANU1,ANU2,G,TT
   20 FORMAT (6F10.0)
      WRITE (3,15) E1,E2,ANU1,ANU2,G,TT
   15 FORMAT (6F16.6)
      PI=3.1416
```

```
      TT=TT*PI/180.
      CC=COS(TT)
      SS=SIN(TT)
      AA=E1/(1.-ANU1*ANU2)
      BB=ANU2*AA
      EE=E2/(1.-ANU1*ANU2)
      DD=G
C     *************************************************************
C     Q(I,J) IS THE STESS-STRAIN CONSITUTIVE MATRIX
C     (T) IS THE TRANSFORMATION MATRIX.
C     Q(N,I,J) IS EQUAL TO (T)T * (Q) * (T) AND IS CALCULATED FOR
C     EVERY LAYER AND STORED.
      Q(N,1,1)=AA*CC**4+2.0*(BB+2.0*DD)*SS**2*CC**2+EE*SS**4
      Q(N,1,2)=(AA+EE-4.0*DD)*SS**2*CC**2+BB*(SS**4+CC**4)
      Q(N,2,2)=AA*SS**4+2.0*(BB+2.0*DD)*SS**2*CC**2+EE*CC**4
      Q(N,1,3)=(AA-BB-2.0*DD)*SS*CC**3+(BB-EE+2.0*DD)*SS**3*CC
      Q(N,2,3)=(AA-BB-2.*DD)*SS**3*CC+(BB-EE+2.*DD)*SS*CC**3
      Q(N,3,3)=(AA+EE-2.*BB-2.*DD)*SS**2*CC**2+DD*(SS**4+CC**4)
      DO 100 I=1,3
      DO 100 J=1,3
  100 Q(N,J,I)=Q(N,I,J)
  101 CONTINUE
C     *************************************************************
C     CALCULATION OF A(I,J) MATRIX WHICH IS THE SUM OF Q(I,J)*
C     H(K) - H(K-1) OVER ALL THE LAYERS.
C     *************************************************************
      DO 200 I=1,3
      DO 200 J=1,3
      A(I,J)=0.0
      DO 180 K=2,KL
  180 A(I,J)=A(I,J)+Q(K,I,J)*(H(K)-H(K-1))
  200 CONTINUE
C     *************************************************************
C     CALCULATION OF B(I,J) MATRIX WHICH IS THE SUM OF Q(I,J)*
C     H(K)**2-H(K-1)**2 * (1/2) OVER ALL THE LAYERS.
C     *************************************************************
      DO 250 I=1,3
      DO 250 J=1,3
      B(I,J)=0.0
      DO 230 K=2,KL
  230 B(I,J)=B(I,J)+Q(K,I,J)*(H(K)**2-H(K-1)**2)*0.5
  250 CONTINUE
C     *************************************************************
C     CALCULATIN OF D(I,J) MATRIX WHICH IS THE SUM OF (1/3) * Q(I,J) *
C     H(K)**3 -H(K-1)**3 OVER ALL THE LAYERS.
C     *************************************************************
      DO 300 I=1,3
      DO 300 J=1,3
      D(I,J)=0.0
      DO 280 K=2,KL
  280 D(I,J)=D(I,J)+Q(K,I,J)*(H(K)**3-H(K-1)**3)/3.
  300 CONTINUE
      DO 305 I=1,3
      DO 305 J=1,3
```

```
      ELN(I,J)=A(I,J)
      ELN(I,J+3)=B(I,J)
      ELN(I+3,J)=ELN(I,J+3)
      ELN(I+3,J+3)=D(I,J)
  305 CONTINUE
      RETURN
      END
/*
//G.FT10F001 DD  UNIT=DISK,SPACE=(CYL,(2,2))
//G.FT11F001 DD  UNIT=DISK,SPACE=(CYL,(2,2))
//G.FT12F001 DD  UNIT=DISK,SPACE=(CYL,(2,2))
//G.FT13F001 DD  UNIT=DISK,SPACE=(CYL,(2,2))
//G.FT14F001 DD  UNIT=DISK,SPACE=(CYL,(2,2))
//G.FT15F001 DD  UNIT=DISK,SPACE=(CYL,(2,2))
//G.FT16F001 DD  UNIT=DISK,SPACE=(CYL,(2,2))
//G.FT17F001 DD  UNIT=DISK,SPACE=(CYL,(2,2))
//G.FT18F001 DD  UNIT=DISK,SPACE=(CYL,(2,2))
//G.FT20F001 DD  UNIT=DISK,SPACE=(CYL,(2,2))
//G.FT21F001 DD  UNIT=DISK,SPACE=(CYL,(2,2))
//G.FT22F001 DD  UNIT=DISK,SPACE=(CYL,(2,2))
//G.FT23F001 DD  UNIT=DISK,SPACE=(CYL,(2,2))
//G.FT24F001 DD  UNIT=DISK,SPACE=(CYL,(2,2))
//G.FT25F001 DD DSN=A.B,DISP=OLD,UNIT=TAPE,
//      VOL=SER=T02246,LABEL=(1,SL,,IN)
//G.SYSIN DD *
```

SAMPLE PROBLEM INPUT DATA



Total Thickness = 0.024 in

Finite-difference grid: 12 x 12

Number of equations: 432

Number of nodal points: 144

Maximum semi-band width: 81

Initial imperfection amplitude: 0.0 in.

Radius of curvature = 25 inches

Boundary Conditions:

Along edge:

$x = 0$ - $w = o, u = o, v = o$

$x = 6$ - $w = o, N_x = o, N x y = o$

$y = 0$ - $w = o, N_y = o, N_{yx} = o$

$y = 4$ - $w = o, N_y = o, N_{yx} = o$

Total no. of layers: 4

Thickness of each layer: 0.006 inches

Fiber orientation in the panel: (+45°, -45°, -45°, +45°)

Modulus of elasticity in fiber direction: 40×10^6 psi

Modulus of elasticity in direction perpendicular to the fiber: 4.5×10^6 psi

Poisson ratio ν_{12} : 0.25

Shearing modulus of elasticity G_{12} : 1.5×10^6 psi

```
    1
*EQUAL MESH PANEL 12 X 12 GRID NO INIT IMPERFECTION**
        144        144     0     0     1
  25.0      0.0001
   0.00      0.0000      0.0000      0.0      0.0
    0     0    12    12     0     0     0     0
    6     2     2     2     2     2     2     2     2     2
    2     7     1     5     5     5     5     5     5     5
    5     5     5     3     1     5     5     5     5     5
    5     5     5     5     5     3     1     5     5     5
    5     5     5     5     5     5     5     3     1     5
    5     5     5     5     5     5     5     5     5     3
    1     5     5     5     5     5     5     5     5     5
    5     3     1     5     5     5     5     5     5     5
    5     5     5     3     1     5     5     5     5     5
    5     5     5     5     5     3     1     5     5     5
    5     5     5     5     5     5     5     3     1     5
    5     5     5     5     5     5     5     5     5     3
    1     5     5     5     5     5     5     5     5     5
    5     3     8     4     4     4     4     4     4     4
    4     4     4     9
    1     2     3     4     5     6     7     8     9    10
   11    12    13    14    15    16    17    18    19    20
   21    22    23    24    25    26    27    28    29    30
   31    32    33    34    35    36    37    38    39    40
   41    42    43    44    45    46    47    48    49    50
   51    52    53    54    55    56    57    58    59    60
   61    62    63    64    65    66    67    68    69    70
   71    72    73    74    75    76    77    78    79    80
   81    82    83    84    85    86    87    88    89    90
   91    92    93    94    95    96    97    98    99   100
  101   102   103   104   105   106   107   108   109   110
  111   112   113   114   115   116   117   118   119   120
  121   122   123   124   125   126   127   128   129   130
  131   132   133   134   135   136   137   138   139   140
  141   142   143   144
    1  2.0         6.07
    2  2.55        6.07
    3  3.10        6.07
    4  3.65        6.07
    5  4.20        6.07
    6  4.75        6.07
    7  5.30        6.07
    8  5.85        6.07
    9  6.40        6.07
   10  6.95        6.07
   11  7.50        6.07
   12  8.05        6.07
   13  2.0         5.700
   14  2.55        5.700
   15  3.10        5.700
   16  3.65        5.700
   17  4.20        5.700
   18  4.75        5.700
```

19	5.30	5.700
20	5.85	5.700
21	6.40	5.700
22	6.95	5.700
23	7.50	5.700
24	8.05	5.700
25	2.0	5.330
26	2.55	5.330
27	3.10	5.330
28	3.65	5.330
29	4.20	5.330
30	4.75	5.330
31	5.30	5.330
32	5.85	5.330
33	6.40	5.330
34	6.95	5.330
35	7.50	5.330
36	8.05	5.330
37	2.0	4.960
38	2.55	4.960
39	3.10	4.960
40	3.65	4.960
41	4.20	4.960
42	4.75	4.960
43	5.30	4.960
44	5.85	4.960
45	6.40	4.960
46	6.95	4.960
47	7.50	4.960
48	8.05	4.960
49	2.0	4.590
50	2.55	4.590
51	3.10	4.590
52	3.65	4.590
53	4.20	4.590
54	4.75	4.590
55	5.30	4.590
56	5.85	4.590
57	6.40	4.590
58	6.95	4.590
59	7.50	4.590
60	8.05	4.590
61	2.0	4.220
62	2.55	4.220
63	3.10	4.220
64	3.65	4.220
65	4.20	4.220
66	4.75	4.220
67	5.30	4.220
68	5.85	4.220
69	6.40	4.220
70	6.95	4.220
71	7.50	4.220
72	8.05	4.220

73	2.0	3.850
74	2.55	3.850
75	3.10	3.850
76	3.65	3.850
77	4.20	3.850
78	4.75	3.850
79	5.30	3.850
80	5.85	3.850
81	6.40	3.850
82	6.95	3.850
83	7.50	3.850
84	8.05	3.850
85	2.0	3.480
86	2.55	3.480
87	3.10	3.480
88	3.65	3.480
89	4.20	3.480
90	4.75	3.480
91	5.30	3.480
92	5.85	3.480
93	6.40	3.480
94	6.95	3.480
95	7.50	3.480
96	8.05	3.480
97	2.0	3.110
98	2.55	3.110
99	3.10	3.110
100	3.65	3.110
101	4.20	3.110
102	4.75	3.110
103	5.30	3.110
104	5.85	3.110
105	6.40	3.110
106	6.95	3.110
107	7.50	3.110
108	8.05	3.110
109	2.0	2.740
110	2.55	2.740
111	3.10	2.740
112	3.65	2.740
113	4.20	2.740
114	4.75	2.740
115	5.30	2.740
116	5.85	2.740
117	6.40	2.740
118	6.95	2.740
119	7.50	2.740
120	8.05	2.740
121	2.0	2.370
122	2.55	2.370
123	3.10	2.370
124	3.65	2.370
125	4.20	2.370
126	4.75	2.370

127	5.30	2.370
128	5.85	2.370
129	6.40	2.370
130	6.95	2.370
131	7.50	2.370
132	8.05	2.370
133	2.0	2.000
134	2.55	2.000
135	3.10	2.000
136	3.65	2.000
137	4.20	2.000
138	4.75	2.000
139	5.30	2.000
140	5.85	2.000
141	6.40	2.000
142	6.95	2.000
143	7.50	2.000
144	8.05	2.000
1	0.0	0.0
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		-50.0
13	0.0	
14	0.0	0.0
15		
16		
17		
18		
19		
20		
21		
22		
23		
24		-50.0
25		
26	0.0	
27	0.0	0.0
28		
29		
30		
31		
32		
33		
34		
35		
36		-50.0

37		
38		
39	0.0	
40	0.0	0.0
41		
42		
43		
44		
45		
46		
47		
48		-50.0
49		
50		
51		
52	0.0	
53	0.0	0.0
54		
55		
56		
57		
58		
59		
60		-50.0
61		
62		
63		
64		
65	0.0	
66	0.0	0.0
67		
68		
69		
70		
71		
72		-50.0
73		
74		
75		
76		
77		
78	0.0	
79	0.0	0.0
80		
81		
82		
83		
84		-50.0
85		
86		
87		
88		
89		
90		

91	0.0	
92	0.0	0.0
93		
94		
95		
96		-50.0
97		
98		
99		
100		
101		
102		
103		
104	0.0	
105	0.0	0.0
106		
107		
108		-50.0
109		
110		
111		
112		
113		
114		
115		
116		
117	0.0	
118	0.0	0.0
119		
120		-50.0
121		
122		
123		
124		
125		
126		
127		
128		
129		
130	0.0	
131	0.0	0.0
132		-50.0
133		
134		
135		
136		
137		
138		
139		
140		
141		
142		
143	0.0	
144	0.0	-50.0

0.0		-20.0	
1	1	1	1
2	1		
3	1		
4	1		
5	1		
6	1		
7	1		
8	1		
9	1		
10	1		
11	1		
12	1	0	1
13	1	1	1
14			
15			
16			
17			
18			
19			
20			
21			
22			
23			
24	1	0	1
25	1	1	1
26			
27			
28			
29			
30			
31			
32			
33			
34			
35			
36	1	0	1
37	1	1	1
38			
39			
40			
41			
42			
43			
44			
45			
46			
47			
48	1	0	1
49	1	1	1
50			
51			
52			
53			

54			
55			
56			
57			
58			
59			
60	1	0	1
61	1	1	1
62			
63			
64			
65			
66			
67			
68			
69			
70			
71			
72	1	0	1
73	1	1	1
74			
75			
76			
77			
78			
79			
80			
81			
82			
83			
84	1	0	1
85	1	1	1
86			
87			
88			
89			
90			
91			
92			
93			
94			
95			
96	1	0	1
97	1	1	1
98			
99			
100			
101			
102			
103			
104			
105			
106			
107			

```
108        1      0      1
109        1      1      1
110
111
112
113
114
115
116
117
118
119
120        1      0      1
121        1      1      1
122
123
124
125
126
127
128
129
130
131
132        1      0      1
133        1      1      1
134        1
135        1
136        1
137        1
138        1
139        1
140        1
141        1
142        1
143        1
144        1      0      1
  4
-0.012
-0.006
 0.0
 0.006
 0.012
40000000.  4500000.  0.25   0.02812   1500000.   +45.0
40000000.  4500000.  0.25   0.02812   1500000.   -45.0
40000000.  4500000.  0.25   0.02812   1500000.   -45.0
40000000.  4500000.  0.25   0.02812   1500000.   +45.0
//
```

REFERENCES

1. Stephen W. Tsia, Mechanics of Composite Materials, Part II-Theoretical Aspects, AFML-TR-66-149, Part II, Air Force Materials Laboratory, Research and Technology Division, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio, November 1966.

2. Robert M. Jones, Mechanics of Composite Materials, McGraw-Hill, 1975.

3. J. E. Ashton and J. M. Whitney, Theory of Laminated Plates, Technomic, Stamford, Conn., 1970.

4. N. S. Khot, On the Effects of Fiber Orientation and Non-homogeneity on Buckling and Postbuckling Equilibrium Behavior of Fiber-Reinforced Cylindrical Shells Under Uniform Axial Compression, Technical Report AFFDL-TR-68-19, Air Force Flight Dynamics Laboratory, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio, May 1968.

5. S. T. Mau and R. H. Gallagher, A Finite Element Procedure for Nonlinear Prebuckling and Initial Postbuckling Analysis, NASA CR-1936, January 1972.

6. R. G. Vos, Finite Element Analysis of Plate Buckling and Post-Buckling, Thesis, Rice University, December 1970.

7. S. Lien, Finite Element Elastic Thin Shell Pre- and Post-Buckling Analysis, Thesis, Cornell University, September 1971.

8. T. E. Lang, Post-Buckling Response of Structures Using the Finite Element Method, Thesis, University of Washington, August 1969.

9. David Bushnell, Analysis of Buckling and Vibration of Ring-Stiffened, Segmented Shells of Revolution, Int. J. Solids and Structures, Vol. 6, 157-181 (1970).

10. D. Bushnell and B. O. Almroth, Finite-Difference Energy Method for Nonlinear Shell Analysis, LMSC/AFFDL Conference on Computer Oriented Analysis of Shell Structures, Palo Alto, California, August 10-14, 1970.

11. David Bushnell, Bo O. Almroth, and Frank Brogan, Finite-Difference Energy Method for Nonlinear Shell Analysis, Jour. Computers and Structures, Vol. 1, 361-387 (1971).

12. B. O. Almroth, F. A. Brogan, M. B. Marlowe, Collapse Analysis for Shells of General Shape, Vol. I Analysis, AFFDL TR-71-8, Air Force Flight Dynamics Laboratory, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio, August 1972.

13. J. R. Tillerson, J. A. Stricklin, and W. E. Haisler, Numerical Methods for the Solution of Nonlinear Problems in Structural Analysis, R. F. Hartung (ed.), "Numerical Solution of Nonlinear Structural Problems," AMD, Vol. 6, ASME, November 1973.

14. Edward L. Wilson, Klaus-Jürgen Bathe, and William P. Doherty, Direct Solutions of Large Systems of Linear Equations, Jour. Computers and Structures, Vol. 4, (363-372) 1974.

15. J. Lyell Sanders, Jr., Nonlinear Theories for Thin Shells, Qtr. of Appl. Math., Vol. XXI, No. 1, 21-36 (1962).

16. Henry L. Langhaar, Energy Methods in Applied Mechanics, Wiley, New York, 1962.

17. B. Budiansky, Dynamic Buckling of Elastic Structures: Criteria and Estimates, Proceedings, International Conference on Dynamic Stability of Structures, Pergamon, New York, 83-106 (1966).

18. B. Budiansky and J. W. Hutchinson, Dynamic Buckling of Imperfection Sensitive Structures, Proceedings of the XI International Congress of Applied Mechanics, edited by H. Gortler, Springer-Verlag, Berlin, 636-651 (1964).

☆U.S.Government Printing Office: 1979 — 657-002/659